Cracking Crypto: An Implied Interest Rate Model for Valuing Cryptocurrency Products

Bitcoin is a peer-to-peer version of electronic cash that allows online payments to be sent directly from one party to another without intermediation of a financial institution. With the rise of global interest in a decentralised ledger, blockchain technology applications such as Bitcoin, have become widely integrated within the financial ecosystem since its introduction 8 years ago. With this recent proliferation, the need for derivative products to hedge volatility in crypto-assets is ultimately growing. Valuing cash flows and hedging products denominated in Bitcoin is difficult due to the lack of a credit market to provide an exogenous interest rate as a parameter to traditional valuation models. This independent study attempts to model pricing contingent claims and cash flows denominated in Bitcoin through application of existing stochastic-jump pricing frameworks. Fieldwork includes a risk analysis project. Read on to know more……

1. Introduction

Currently, Bitcoin is recognised as a possible alternative, geopolitically neutral currency and is classified as one of the first digital currencies, or cryptocurrencies. Although cryptocurrencies and related financial products have been known for obscure speculative purposes and grey market transactions, the growing mainstream integration of these products creates the need for asset stability as an investment vehicle and as a commercial transactional ledger. Cryptocurrency assets are at the nascent stage of their life cycles, so the need for traditional hedging instruments overlaying these assets to hedge volatility is needed. Due to market uncertainty and regulation, market making activity and underwriting such derivatives has been scarce in the United States and internationally. Another obstacle in the process of providing stability for cryptomarkets is finding the present

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value of future or past cash flows and valuing derivative products with such digital assets as the underlier. Although the infrastructure surrounding cryptocurrency derivatives market making presents an enormous obstacle, this paper attempts to address the secondary problem of pricing theoretical cryptocurrency derivatives.

Given the current problem state, this study examines, more specifically, pricing Non-Deliverable Forwards and Futures ("NDFs") of Bitcoin currency pairs since Bitcoin is the most liquid cryptocurrency. All data presented uses a weighted average of multiple exchange rates for Bitcoin/US Dollar ("BTC/USD") as the underlier. Included in this article is a project valuation and risk analysis of Bitcoin transactions and a proposed application of an implied interest rate model for pricing derivatives and valuing future Bitcoin denominated cash flows.

2. Blockchain and Bitcoin

In 2009, an unknown group of coders (under the pseudonym of "Satoshi Nakamoto") introduced Bitcoin to the world as a peer-to-peer ("P2P") open-source software that operates to create and maintain a distributed public ledger. Bitcoin is based on the underlying "blockchain technology" which is analogous to a database that maintains the record of all the changes made to it since its creation through mutual consensus of its users. This blockchain keeps growing as newly completed blocks (i.e. transactions) are added. The blocks are linked to each other, like a chain, in a proper linear, chronological order with every block containing a hash of the previous block. The blockchain is decentralised and is maintained by a network of computers referred to as "miners" who solve complex computationally intensive exercises for financial reward as part of verifying amendments to the ledger and maintaining the network. The main characteristics of Bitcoin are:

1. Decentralisation: Unlike traditional currency there is no central authority to regulate its supply or manipulate its value. The value of Bitcoin fluctuates primarily due to its fiat market value
2. Cryptographic Architecture: Bitcoin transactions are validated on a P2P basis in a decentralised manner
3. No Third Party: Unlike earlier version of digital currencies, Bitcoin does not need the trusted third party to validate transactions.

The transaction process for the functioning and addition of new blocks to the blockchain is summarised in Figure 1 below:

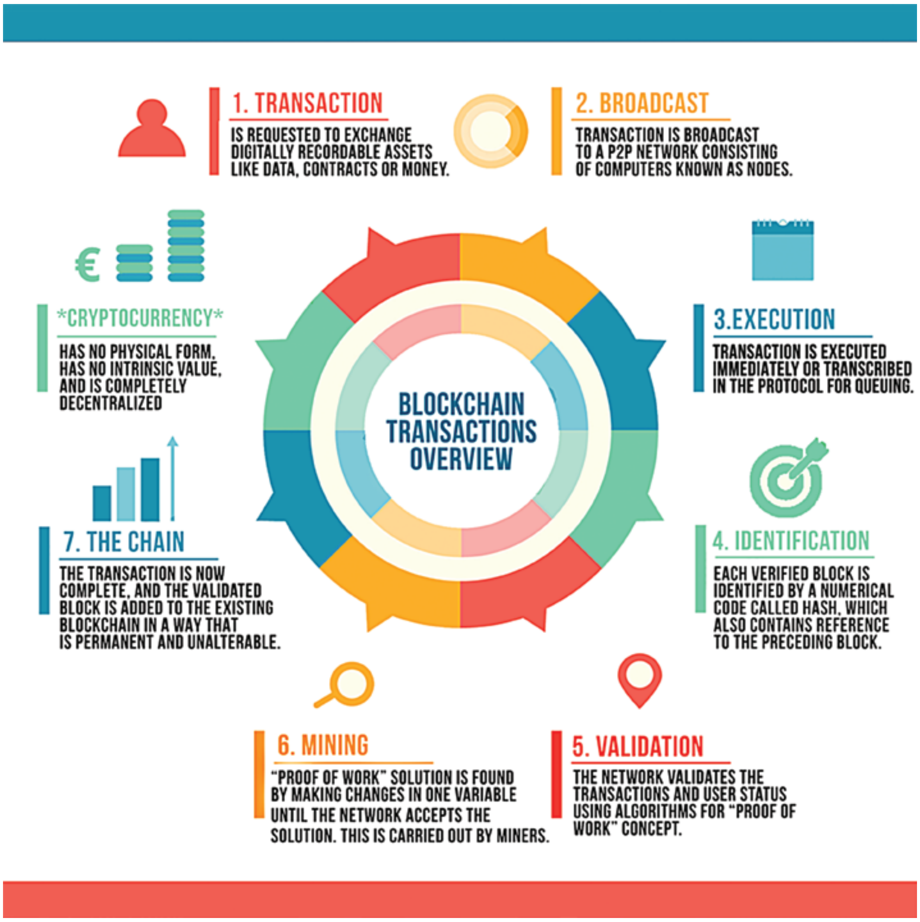

Figure 1: Blockchain Transaction Overview

3. Bitcoin Derivatives

A derivative is a contract between two or more parties whose value is based on an agreed-upon underlying financial asset, index, or security. Forwards, Futures, Options and Swaps are the most common examples of derivative instruments.

In the United States, most derivative contracts are subject to regulation by the Commodity Futures Trading Commission ("CFTC") which regulates futures and swaps markets to protect participants from fraud, market manipulation, abusive practices, and systemic risk. Since the inception of Bitcoin in 2009, there have been few CFTC recognised Swap Execution Facilities ("SEFs") for cryptographic assets. However, with growing interest in Bitcoin and blockchain technology, the need for derivatives products to hedge volatility, and transact in the currency is leading to the growth of these products.

A fundamental and deeply developed derivative product is the Non-Deliverable Forward which is an outright forward or futures contract in which counterparties settle the difference between the contracted NDF price or rate and the prevailing spot price or rate on an agreed notional amount. NDFs are priced using the following formula given in (Equation 1):

$$F = \frac{S \times (1 + Rh \times t)}{(1 + Rf \times t)}$$

Equation 1: Non-Deliverable Forward Price

Where, F is the price, S is the spot price in BTC/USD, Rh is the home interest rate or U.S. Treasury rates, *Rf* is the foreign interest rate which is to be solved, and t is time frame in years (the number of days divided by 365).

4. Implied Interest Rate Model

The value of Bitcoin/US Dollar (BTC/USD) and other cross pairs is very volatile, subject to media perceptions, macroeconomic conditions, and dynamic technical factors of the blockchain (i.e. memory pool size changes, fork risks, or consensus attacks). The need to hedge volatility in Bitcoin is paramount for participants, but two primary obstacles stifle progress in offering an adoptable solution:

1. The infrastructure for offering derivative products is obscure and often scrutinised by U.S. regulators.
2. The lack of a Bitcoin credit market makes valuing derivative products and cash flows denominated in BTC difficult due to the lack of a known, exogenous interest rate.

Upon further examination of the Bitcoin monetary system, it may be deduced that an implied interest rate, comprised of a deterministic inflation factor and stochastic "residual premium," can be used as a proxy discount rate for valuation and derivatives pricing. This implied interest rate (r) is composed of:

1. The deterministic "inflation factor" (*i*), which represents the intrinsic growth rate of the number of Bitcoins created over time.

2. The stochastic "residual premium" (ρ), which captures the market demand effects of Bitcoin with respect to its cross pair.

5. Supply Side: The deterministic inflation factor (i)

Bitcoins are created, or mined, by the nodes of the peer-to-peer network and the Bitcoin generation algorithm defines, in advance, how currency will be created and at what rate. Further, any currency that is generated by a malicious user that does not follow the rules will be rejected by the network and thus is worthless, offering security and concurrency. Therefore, without a central bank regulating monetary supply, Bitcoin is subject to a controlled supply via a decentralised network and the inflation rate of Bitcoin (i), unlike that of other fiat currencies, is deterministic. Moreover, since the supply is regulated, the number of Bitcoins generated per block is set to decrease geometrically, using a 50% reduction for every 210,000 blocks, or approximately every four years. The result is that the number of Bitcoins in existence is not expected to exceed 21,000,000 coins in circulation at any point in time.

This deterministic relationship between the number of Bitcoins in the network and time allows the observance of an inflation rate for Bitcoin. The inflation rate could be defined by the content of the 'input' of a generation transaction as a product of blocks mined per year with respect to existing supply of coins (Equation 2):

$$i = CoinBase * \frac{BlocksPerYear}{ExistingCoins}$$

Equation 2: Inflation Rate of Bitcoin

Since the number of Bitcoins added to the chain is constant, as the number of Bitcoins grow, the constant number of added Bitcoins becomes relatively smaller compared to the total number of Bitcoins, causing the inflation rate of Bitcoin to approach zero. As of July 2017, the Bitcoin inflation rate is roughly 7%.

6. Demand Side: The stochastic residual premium (ρ)

The value of Bitcoin is not solely attributed to the intrinsic inflation rate, or supply-side effects, value is also affected by its market demand. These "demand side" effects have been defined as the residual effects of Bitcoin's value that are not intrinsically tied to its deterministic monetary supply.

This residual factor contains the effects beyond supply changes in Bitcoin such as geopolitical changes, media perceptions, hacking, and other material events that affect Bitcoin or currency cross pair valuation. News and events regarding this developing cryptocurrency can thus have a large effect on the current value. These kinds of abrupt changes are certainly hard to predict, but are quite common in the burgeoning market. Thus, to model the cash flows and accurately price Bitcoin denominated derivatives, a residual risk "premium" is used to not only account for the growth of Bitcoin as random Brownian motion, but also to incorporate a jump process, which attempts to model occasional value dispersing events.

The premium above or below the inflation rate, or supply side of Bitcoin value, is the residual premium (ρ) which represents the demand effects of Bitcoin valuation.

The residual premium model attempts to capture the empirical results of implied rates and the ability to achieve a closed-form solution. Robert Merton's Option Pricing when *Underlying Stock Returns are Discontinuous* (1976) was the first to propose that underlying asset returns are generated by a mixture of both continuous and jump processes. Jump-diffusion models do not make the same assumption as the Black Scholes model of normally distributed logarithmic returns from the underlying asset. This allows for the asset return model to capture empirical results when an asset does not follow definitional geometric Brownian motion. Through backtesting, it was observed that the implied interest rate of Bitcoin has periods of stable diffusion processes, appearing to follow standard geometric Brownian motion, as well as short periods of large positive and negative "jumps." This empirical behaviour justifies the use case for a jump-diffusion process for Bitcoin.

Although Merton's model is the first important foray into jump-diffusion in options pricing, Dr. Steven Kou's double exponential jump diffusion model has been applied due to its ability to obtain closed-form solutions via the memoryless property of the exponential and empirical properties to reflect over- and under-reactions to outside news via Jump processes.

The heavy tails caused by the double exponential distribution in the exponent captures these large "demand effect" changes, or market spikes and drops. As with the Merton model, the stock price consists of two parts: a continuous factor driven by a standard geometric Brownian motion and a jump factor with a logarithm of jump size, which is doubly exponentially distributed. The number of jumps is determined by the event times of a Poisson process, which was assumed to be 36-month average arrival times in displayed results. Equations 4 and 5 apply the Kou jump-diffusion framework to our residual premium factor in stochastic differential and closed forms. This demand factor ρ plus the inflation factor i gives the overall implied interest rate *Rf* that is used in Equation 1 to price Bitcoin NDFs and used to get the present value of Bitcoin denominated cash flows. Equation 4 and 5 are the differential and closed form equations for the stochastic demand process.

$$\frac{\mathrm{d}\rho(t)}{\rho(t)} = \mu dt + \sigma dW(t) + d\left(\sum_{i=1}^{N(t)} (V_i - 1)\right)$$

Equation 4: Residual Premium Differential Equation

$$\rho(t) = \rho(0) exp\left\{\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma W(t)\right\}\prod_{i=1}^{N(t)} V_i$$

Equation 5: Residual Premium Closed Form Equation

7. Appropriateness of the Demand Side Mode

The observed average historical BTC.USD exchange rates calculated by weighted averaging prices by market share from three major Bitcoin exchanges: BTC-e, BitStamp, and BitFinex are depicted in Figure 3 which satisfies Equation 1 for forward prices.

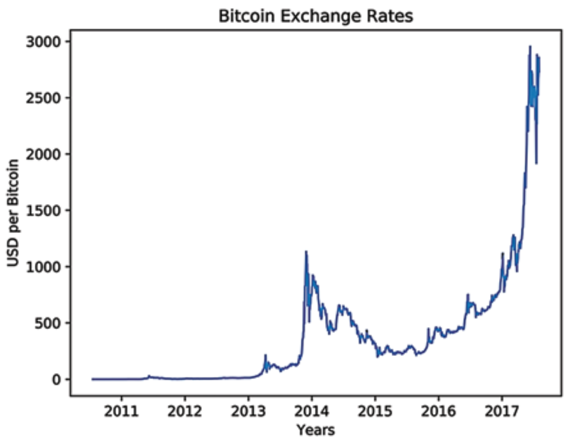

Figure 3: Historical BTC/USD exchange rates

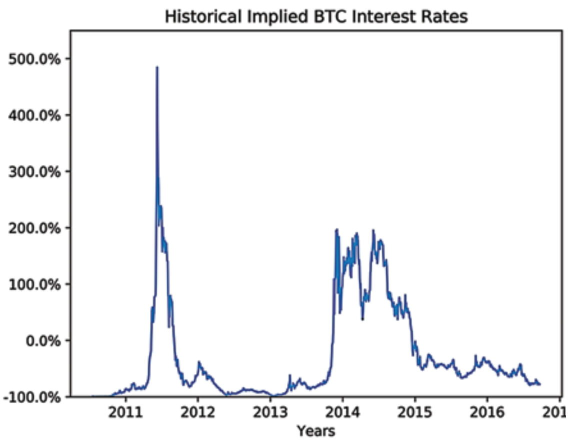

Figure 4: Historical Implied BTC Interest Rates Obtained from Model

Further, the implied rates are observed in Figure 4, showing evolution over the sample period January 2011 to January 2017. In this model, the residual demand is constructed in such a way to explain the large jumps and relatively stable diffusion periods seen in the historical implied Bitcoin interest rates.

Stable Pattern: As seen in the graph, the interest rates follow a relatively stable pattern from Q2 2010 to Q2 2011, from Q1 2012 to Q3 2013, and from Q1 2015 to approximately Q2 2016. In addition, there is also relative stability between 2014 and 2015, where fluctuations around 150% occur. These relatively stable regions follow geometric Brownian motion.

Unstable Pattern: Large positive and negative jumps are seen around mid-2011, positive jumps in the beginning of 2014, and negative jumps in Q3 2014 and Q1 2015. To justify these very large jumps, the residual premium model uses the double exponential process to capture the effect of heavy tails.

8. Conclusion

The increasing popularity and growing usage of digital currencies, especially Bitcoin coupled with inherent volatility created by daily fluctuations resulting from different risks of geography, politics, and security creates a need for hedging products with these currencies as an underlying instrument. Hence, it is vital to understand Bitcoin's fluctuations and to be able to predict the value of future cash flows in order to understand the measure of risks and protect against them.

To solve these issues, an implied interest rate model has been proposed to replicate interest rates and resolve the issues in valuing the Bitcoin derivatives due to the lack of credit market. The model was also tested on historical data to obtain past implied rates, and a clear trend was observed, with periods of stability interjected by sharp volatile spikes accounted for by the stochastic jumps.

9. Selected References

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To succeed in your mission, you must have single-minded devotion to your goal. - Abdul Kalam