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Time Value of Money

BASIC CONCEPTS AND FORMULAE

1. Time Value of Money	It means money has time value. A rupee today is more valuable than a rupee a year hence. We use rate of interest to express the time value of money.
2. Simple Interest	Simple Interest may be defined as Interest that is calculated as a simple percentage of the original principal amount. <ul style="list-style-type: none"> • Formula for Simple Interest $SI = P_0 (i)(n)$
3. Compound Interest	Compound interest is the interest calculated on total of previously earned interest and the original principal. <ul style="list-style-type: none"> • Formula for Compound Interest $FV_n = P_0 (1+i)^n$
4. Present Value of a Sum of Money	Present value of a sum of money to be received at a future date is determined by discounting the future value at the interest rate that the money could earn over the period.
5. Future Value	Future Value is the value at some future time of a present amount of money, or a series of payments, evaluated at a given interest rate. <ul style="list-style-type: none"> ❖ Formula for Future Value $FV_n = P_0 + SI = P_0 + P_0(i)(n)$ Or, $FV_n = P_0 \left(1 + \frac{r}{k}\right)^n$
6. Annuity	An annuity is a series of equal payments or receipts occurring over a specified number of periods. <ul style="list-style-type: none"> • Present Value of an Ordinary Annuity: Cash flows occur at the end of each period, and present value is calculated as of one period before the first cash flow.

	<ul style="list-style-type: none"> • Present Value of an Annuity Due: Cash flows occur at the beginning of each period, and present value is calculated as of the first cash flow. <ul style="list-style-type: none"> ❖ Formula for Present Value of An Annuity Due $PV_{An} = R (PVIF_{i,n})$ • Future Value of an Ordinary Annuity: Cash flows occur at the end of each period, and future value is calculated as of the last cash flow. • Future Value of an Annuity Due: Cash flows occur at the beginning of each period, and future value is calculated as of one period after the last cash flow. <ul style="list-style-type: none"> ❖ Formula for Future Value of an Annuity Due $FV_{An} = R (FVIFA_{i,n})$
7. Sinking Fund	<p>It is the fund created for a specified purpose by way of sequence of periodic payments over a time period at a specified interest rate.</p> <ul style="list-style-type: none"> ❖ Formula for Sinking Fund $FVA = R [FVIFA(i,n)]$

SECTION-A

Question 1

Explain the relevance of time value of money in financial decisions. Or

Why money in the future is worth less than similar money today? Give reasons and explain.

Answer

Time value of money means that worth of a rupee received today is different from the worth of a rupee to be received in future. The preference of money now as compared to future money is known as time preference for money.

A rupee today is more valuable than rupee after a year due to several reasons:

- Risk – there is uncertainty about the receipt of money in future.
- Preference for present consumption – Most of the persons and companies in general, prefer current consumption over future consumption.
- Inflation – In an inflationary period a rupee today represents a greater real purchasing power than a rupee a year hence.

2.3 Financial Management

- Investment opportunities – Most of the persons and companies have a preference for present money because of availabilities of opportunities of investment for earning additional cash flow.

Many financial problems involve cash flow accruing at different points of time for evaluating such cash flow an explicit consideration of time value of money is required.

Question 2

Define 'Present Value' and 'Perpetuity'.

Answer

Present Value: Present Value” is the current value of a “Future Amount”. It can also be defined as the amount to be invested today (Present Value) at a given rate over specified period to equal the “Future Amount”.

Perpetuity: Perpetuity is an annuity in which the periodic payments or receipts begin on a fixed date and continue indefinitely or perpetually. Fixed coupon payments on permanently invested (irredeemable) sums of money are prime examples of perpetuities.

Question 3

Explain:

- (i) Time value of money;
- (ii) Simple interest and
- (iii) Compound interest

Answer

(i) **Time Value of Money:** It means money has time value. A Rupee today is more valuable than a rupee a year hence. We use rate of interest to express the time value of money.

(ii) **Simple Interest:** Simple Interest may be defined as interest that is calculated as a simple percentage of the original principal amount.

Formula is $SI = P_0 (i) (n)$

(iii) **Compound Interest:** Compound Interest is the interest calculated on total of previously earned interest and the original principal.

Formula is $FV_n = P_0 (1 + i)^n$

SECTION-B

Question 1

Calculate if ₹ 10,000 is invested at interest rate of 12% per annum, what is the amount after 3 years if the compounding of interest is done?

- (i) Annually
(ii) Semi-annually
(iii) Quarterly

Answer

Computation of future value

Principal (P_0) = ₹ 10,000

Rate of interest (i) = 12% p.a.

Time period (n) = 3 years

Amount if compounding is done:

(i) Annually

$$\begin{aligned} \text{Future Value} &= P(1+i)^n \\ &= ₹10,000 (1 + 0.12)^3 \\ &= ₹10,000 \times 1.404928 \\ &= ₹ 14,049.28 \end{aligned}$$

(ii) Semi Annually

$$\begin{aligned} \text{Future Value} &= ₹10,000 \left(1 + \frac{12}{100 \times 2}\right)^{3 \times 2} \\ &= ₹10,000 (1 + 0.06)^6 \\ &= ₹10,000 \times 1.418519 \\ &= ₹ 14,185.19 \end{aligned}$$

(iii) Quarterly

$$\begin{aligned} \text{Future Value} &= ₹10,000 \left(1 + \frac{12}{100 \times 2}\right)^{3 \times 4} \\ &= ₹10,000 (1 + 0.03)^{12} \\ &= ₹10,000 \times 1.425761 \\ &= ₹14,257.61 \end{aligned}$$

Question 2

A person is required to pay four equal annual payments of ₹ 4,000 each in his Deposit account that pays 10 per cent interest per year. Find out the future value of annuity at the end of 4 years.

2.5 Financial Management

Answer

$$FVA = A \left(\frac{(1+i)^n - 1}{i} \right) = ₹ 4,000 \left(\frac{(1+0.10)^4 - 1}{0.10} \right) = ₹ 4,000 \times 4.641 = ₹ 18,564$$

Future Value of Annuity at the end of 4 years = ₹ 18,564

Question 3

A company offers a fixed deposit scheme whereby ₹ 10,000 matures to ₹ 12,625 after 2 years, on a half-yearly compounding basis. If the company wishes to amend the scheme by compounding interest every quarter, what will be the revised maturity value?

Answer

Computation of Rate of Interest and Revised Maturity Value

Principal = ₹ 10,000

Amount = ₹ 12,625

$$10,000 = \frac{₹12,625}{(1+i)^4}$$

$$P_n = A \times (PVF_{n,i})$$

$$₹ 10,000 = 12,625 (PVF_{4,i})$$

$$0.7921 = (PVF_{4,i})$$

According to the Table on Present Value Factor ($PVF_{4,i}$) of a lump sum of ₹1, a PVF of 0.7921 for half year at interest (i) = 6 percent. Therefore, the annual interest rate is $2 \times 0.06 = 12$ percent.

$$i = 6\% \text{ for half year}$$

$$i = 12\% \text{ for full year.}$$

Therefore, Rate of Interest = 12% per annum

$$\begin{aligned} \text{Revised Maturity Value} &= 10,000 \left(1 + \frac{12}{100} \times \frac{1}{4} \right)^{2 \times 4} = 10,000 \left(1 + \frac{3}{100} \right)^8 = 10,000 (1.03)^8 \\ &= 10,000 \times 1.267 \text{ [Considering } (CVF_{8,3}) = 1.267] \end{aligned}$$

Revised Maturity Value = ₹ 12,670

Question 4

A doctor is planning to buy an X-Ray machine for his hospital. He has two options. He can either purchase it by making a cash payment of ₹ 5 lakhs or ₹ 6,15,000 are to be paid in six equal annual installments. Which option do you suggest to the doctor assuming the rate of return is

12 percent? Present value of annuity of Re. 1 at 12 percent rate of discount for six years is 4.111.

Answer

Option I: Cash Down Payment

Cash down payment = ₹ 5,00,000

Option II: Annual Installment Basis

Annual installment = ₹ 6,15,000 × $\frac{1}{6}$ = ₹ 1,02,500

Present Value of 1 to 6 instalments @12%

= ₹1,02,500 × 4.111

= ₹ 4,21,378

Advise: The doctor should buy X-Ray machine on installment basis because the present value of cash out flows is lower than cash down payment. This means Option II is better than Option I.

Question 5

Ascertain the compound value and compound interest of an amount of ₹ 75,000 at 8 percent compounded semiannually for 5 years.

Answer

Computation of Compound Value and Compound Interest

Semiannual Rate of Interest (i) = $8/2 = 4\%$

$n = 5 \times 2 = 10$, $P = ₹ 75,000$

Compound Value = $P(1+i)^n$
 = 75,000 (1+4%)¹⁰
 = 75,000 × 1.4802
 = ₹ 1,11,015

Compound Interest = ₹ 1,11,015 – ₹ 75,000 = ₹ 36,015

Question 6

X is invested ₹ 2,40,000 at annual rate of interest of 10 percent. What is the amount after 3 years if the compounding is done?

(i) *Annually*

(ii) *Semi-annually.*

2.7 Financial Management

Answer

Computation of Future Value

Principal (P) = ₹ 2,40,000

Rate of Interest (i) = 10% p.a.

Time period (n) = 3 years

Amount if compounding is done:

(i) Annually

$$\text{Future Value} = P (1 + i)^n$$

$$= ₹ 2,40,000 \left(1 + \left(1 + \frac{10}{100}\right)^3\right) = ₹ 2,40,000 (1 + 0.1)^3 = ₹ 2,40,000 \times 1.331 = ₹ 3,19,440$$

(ii) Semi-Annually

$$\text{Future Value} = 2,40,000 \left(1 + \frac{10}{100 \times 2}\right)^{3 \times 2}$$

$$= ₹ 2,40,000 (1 + 0.05)^6$$

$$= ₹ 2,40,000 \times (1.05)^6$$

$$= ₹ 2,40,000 \times 1.3401$$

$$= ₹ 3,21,624$$