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## Time Value of Money

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### Learning Objectives

After studying this chapter you will be able to:

- ◆ Understand the Concept of time value of money.
- ◆ Understand the relationship between present and future value of money and how interest rate is used to adjust the value of cash flows in-order to arrive at present (discounting) or future (compounding) values.
- ◆ Understand how to calculate the present or future value of an annuity?
- ◆ Know how to use interest factor table's in order to calculate the present or future values?

### Overview

This chapter basically tries to impart you the concept and importance of monies worth today as compared to in the future. It talks about present value and future value of your money or investment. It discusses the concept of opportunity cost and the importance to know how to compute the time value of money so that you can distinguish between the worth of investments that offer you returns at different times. This chapter is of utmost importance as other chapters will expand on the concepts learnt in this chapter. For instance, time value concept forms the basis of all the modern tools and techniques of capital budgeting decisions like net present value (NPV) method, internal rate of return method (IRR) to name a few dealt in Chapter Six under Investment Decisions.

### 2.1 Concept of Time Value of Money

Let's start a discussion on Time Value of Money by taking a very simple scenario. If you are offered the choice between having ₹ 10,000 today and having ₹ 10,000 at a future date, you will usually prefer to have ₹ 10,000 now. Similarly, if the choice is between paying ₹ 10,000 now or paying the same ₹ 10,000 at a future date, you will usually prefer to pay ₹ 10,000 later. It is simple common sense. In the first case by accepting ₹ 10,000 early, you can simply put the money in the bank and earn some interest. Similarly in the second case by deferring the payment, you can earn interest by keeping the money in the bank.

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Therefore the time gap allowed helps us to make some money. This incremental gain is time value of money.

Now let me ask a question, if the bank interest was zero (which is generally not the case), what would be the time value of money? As you rightly guessed it would also be zero.

As we understood above, the interest plays an important role in determining the time value of money. Interest rate is the cost of borrowing money as a yearly percentage. For investors, interest rate is the rate earned on an investment as a yearly percentage.

### 2.2 Reasons Why Money in the Future is Worth Less Than Similar Money Today

There are three reasons why money can be more valuable today than in the future. Let's discuss them:

(i) **Preference for Present Consumption:** Individuals have a preference for current consumption in comparison to future consumption. In order to forego the present consumption for a future one, they need a strong incentive. Say for example, if the individual's present preference is very strong then he has to be offered a very high incentive to forego it like a higher rate of interest and vice versa.

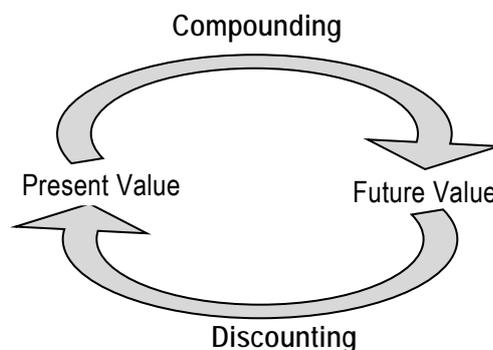
(ii) **Inflation:** Inflation means when prices of things rise faster than they actually should. When there is inflation, the value of currency decreases over time. If the inflation is more, then the gap between the value of money today to the value of money in future is more. So, greater the inflation, greater is the gap and vice versa.

(iii) **Risk:** Risk of uncertainty in the future lowers the value of money. Say for example, non-receipt of payment, uncertainty of investor's life or any other contingency which may result in non-payment or reduction in payment.

Time value of money results from the concept of interest. So it is now time to discuss Interest.

### 2.3 Compounding and Discounting

Compounding is the process of calculating future values of cash flows where discounting means finding present value of cash flows.



## 2.4 Simple Interest & Compound Interest

**2.4.1 Simple Interest:** It may be defined as Interest that is calculated as a simple percentage of the original principal amount. Please note the word “Original”. The formula for calculating simple interest is:

$$SI = P_0 (i)(n)$$

Where,

SI = simple interest in rupees

$P_0$  = original principal

$i$  = interest rate per time period (in decimals)

$n$  = number of time periods

If we add principal to the interest i.e.  $P_0 + P_0 (i)(n)$ , we will get the total future value (FV).

**2.4.2 Compound Interest:** If interest is calculated on original principal amount it is simple interest. When interest is calculated on total of previously earned interest and the original principal it compound interest. Naturally, the amount calculated on the basis of compound interest rate is higher than when calculated with the simple rate.

### The Magic of Compound Interest – Rule of 72

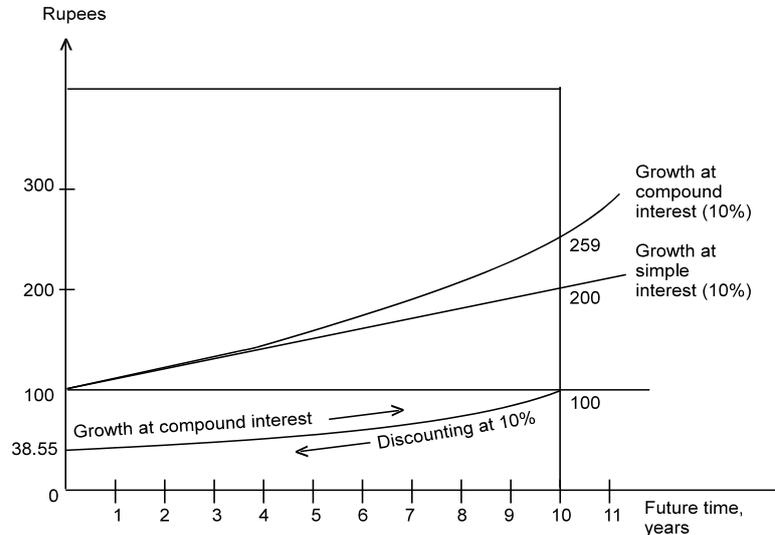
*(It depicts the effect of compounding ₹ 1,000 lump sum at various ages and interest rates).*

	Age of an Individual	Interest Rate						
Divide 72 by the interest rate or inflation rate to estimate the number of years it takes for your money to double for or against you.		4%		6%		8%		12%
	25	1,000	25	1,000	25	1,000	25	1,000
	43	2,000	37	2,000	34	2,000	31	2,000
	61	4,000	49	4,000	43	4,000	37	4,000
	79	8,000	61	8,000	52	8,000	43	8,000
			73	16,000	61	16,000	49	16,000
			85	32,000	70	32,000	55	32,000
					79	64,000	61	64,000
							67	1,28,000
						73	2,56,000	
						79	5,12,000	

**2.4.3 Compound Interest versus Simple Interest:** The given figure shows graphically the differentiation between compound interest and simple interest. The top two ascending lines show the growth of ₹ 100 invested at simple and compound interest. The longer the

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funds are invested, the greater the advantage with compound interest. The bottom line shows that ₹ 38.55 must be invested now to obtain ₹ 100 after 10 periods. Conversely, the present value of ₹ 100 to be received after 10 years is ₹ 38.55.



Compound Interest versus Simple Interest

**2.4.4 Future Value:** This is also known as terminal value. The accrued amount  $FV_n$  on a principal  $P$  after  $n$  payment periods at  $i$  (in decimal) rate of interest per payment period is given by:

$$FV_n = P_0 (1 + i)^n,$$

Where,

$$i = \frac{\text{Annual rate of interest}}{\text{Number of payment periods per year}} = \frac{r}{k}.$$

$(1 + i)^n$  is known as future value factor or compound value factor.

So  $FV_n = P_0 \left(1 + \frac{r}{k}\right)^n$ , when compounding is done  $k$  times a year at an annual interest rate  $r$ .

Or

$$FV_n = P_0 (FVIF_{i,n}),$$

Where,

$FVIF_{i,n}$  is the future value interest factor at  $i\%$  for  $n$  periods equal  $(1 + i)^n$ .

Computation of  $FV_n$  shall be quite simple with a calculator. However, compound interest tables as well as tables for  $(1+i)^n$  at various rates per annum with (a) annual compounding; (b) monthly compounded and (c) daily compounding are available.

**Illustration 1:** Determine the compound interest for an investment of ₹ 7,500 at 6 % compounded half-yearly. Given that  $(1+i)^n$  for  $i = 0.03$  and  $n = 12$  is 1.42576.

**Solution**

$$i = \frac{6}{2 \times 100} = 0.03, \quad n = 6 \times 2 = 12, \quad P = 7,500$$

$$\text{Compound Amount} = 7,500(1+0.03)^{12} = 7,500 \times 1.42576 = 10,693.20$$

$$\text{Compound Interest} = 10,693.20 - 7,500 = 3,193.20$$

**Illustration 2:** ₹ 2,000 is invested at annual rate of interest of 10%. What is the amount after 2 years if the compounding is done?

- (a) Annually?      (b) Semi annually?      (c) Monthly?      (d) Daily?

**Solution**

- (a) The annual compounding is given by:

$$\begin{aligned} \text{FV}_2 &= P(1+i)^n, \text{ n being 2, i being } \frac{10}{100} = 0.1 \text{ and P being 2,000} \\ &= 2,000(1.1)^2 = 2,000 \times 1.21 = ₹ 2,420 \end{aligned}$$

- (b) For Semiannual compounding,  $n = 2 \times 2 = 4$ ,  $i = 0.1/2 = 0.05$

$$\text{FV}_4 = 2,000(1+0.05)^4 = 2,000 \times 1.2155 = ₹ 2,431$$

- (c) For monthly compounding,  $n = 12 \times 2 = 24$ ,  $i = 0.1/12 = 0.00833$

$$\text{FV}_{24} = 2,000(1.00833)^{24} = 2,000 \times 1.22029 = ₹ 2,440.58$$

- (d) For daily compounding,  $n = 365 \times 2 = 730$ ,  $i = 0.1/(365) = 0.00027$

$$\text{FV}_{730} = 2,000(1.00027)^{730} = 2,000 \times 1.22135 = ₹ 2,442.70$$

**Illustration 3:** Determine the compound amount and compound interest on ₹ 1,000 at 6% compounded semiannually for 6 years. Given that  $(1+i)^n = 1.42576$  for  $i = 3\%$  and  $n = 12$ .

**Solution**

$$i = (6/2) = 3\%, \quad n = 6 \times 2 = 12, \quad P = 1,000$$

$$\begin{aligned} \text{Compound amount} &= P(1+i)^n = 1,000(1+3\%)^{12} \\ &= 1,000 \times 1.42576 = ₹ 1,425.76 \end{aligned}$$

$$\text{Compound interest} = 1,425.76 - 1,000 = ₹ 425.76$$

**Illustration 4:** What annual rate of interest compounded annually doubles an investment in 7 years? Given that  $2^{1/7} = 1.104090$ .

**Solution**

$$\text{If the principal be P, } \text{FV}_n = 2P$$

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Since,  $FV_n = P(1 + i)^n$ ,

$$2P = P(1 + i)^7,$$

$$\text{Or, } 2 = (1 + i)^7$$

$$\text{Or, } 2^{1/7} = 1 + i$$

$$\text{Or, } 1.104090 = 1 + i \quad \text{i.e., } i = 0.10409$$

Required rate of interest = 10.41%

**Illustration 5:** A person opened an account on April, 2012 with a deposit of ₹ 800. The account paid 6% interest compounded quarterly. On October 1, 2012, he closed the account and added enough additional money to invest in a 6-month Time Deposit for ₹ 1,000 earning 6% compounded monthly.

- How much additional amount did the person invest on October 1?
- What was the maturity value of his Time Deposit on April 1, 2013?
- How much total interest was earned?

Given that  $(1 + i)^n$  is 1.03022500 for  $i = 1\frac{1}{2}\%$ ,  $n = 2$  and is 1.03037751 for  $i = \frac{1}{2}\%$  and  $n = 6$ .

### Solution

- The initial investment earned interests for April – June and July – September quarter, i.e. for 2 quarters.

$$\text{In this case, } i = \frac{6}{4} = 1\frac{1}{2}\%, n = 2 \text{ and the compounded amount} = 800 \left(1 + 1\frac{1}{2}\%\right)^2$$

$$= 800 \times 1.03022500 = ₹ 824.18$$

$$\text{The additional amount} = ₹ (1,000 - 824.18) = ₹ 175.82$$

- In this case, the Time Deposit earned interest compounded monthly for 2 quarters.

$$\text{Here, } i = \frac{6}{12} = \frac{1}{2}\%, n = 6, P = 1,000$$

$$\text{Required maturity value } 1,000 \left(1 + \frac{1}{2}\%\right)^6 = 1,000 \times 1.03037751 = ₹ 1,030.38$$

- Total interest earned =  $(24.18 + 30.38) = ₹ 54.56$

**Illustration 6:** Ramanuj has taken a 20 month car loan of ₹ 6,00,000. The rate of interest is 12 per cent per annum. What will be the amount of monthly loan amortization?

**Solution**

$$A = \frac{₹ 6,00,000}{PVIFA_{1,20}} = \frac{₹ 6,00,000}{18.0456} = ₹ 33,249.1$$

Monthly interest = 12 per cent/12 = 1 per cent.

## 2.5 Effective Rate of Interest (EIR)

It is the actual equivalent annual rate of interest at which an investment grows in value when interest is credited more often than once a year. If interest is paid  $m$  times in a year it can be found by calculating:

$$E_i = \left(1 + \frac{i}{m}\right)^m - 1$$

**Illustration 7:** If the interest is 10% payable quarterly, find the effective rate of interest.

**Solution**

$$E = \left(1 + \frac{0.1}{4}\right)^4 - 1 = 0.1038 \text{ or } 10.38\%$$

**2.5.1 Multi-period Compounding:** In case of multi period compounding it can be compounded as below:

Conversion Period	Description
1 day	Compounded daily
1 month	Compounded monthly
3 months	Compounded quarterly
6 months	Compounded semiannually
12 months	Compounded annually

The general formula of effective interest rate shall be

$$EIR = \left(1 + \frac{i}{m}\right)^{n \times m} - 1$$

Daily compounding is also known as continuous compounding.

Effective interest rate can be calculated as

$$EIR = \left(1 + \frac{i}{365}\right)^{1 \times 365} - 1$$

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$$\text{Or, } FV_n = P \times e^{(i \times n)} = P \times e^x$$

$$x = (i \times n)$$

$$e = 2.7183$$

### 2.6 Present Value

Let's first define Present Value. Simple definition is "Present Value" is the current value of a "Future Amount". It can also be defined as the amount to be invested today (Present Value) at a given rate over specified period to equal the "Future Amount".

If we reverse the flow by saying that we expect a fixed amount after n number of years, and we also know the current prevailing interest rate, then by discounting the future amount, at the given interest rate, we will get the present value of investment to be made.

Discounting future amount converts it into present value amount. Similarly, compounding converts present value amount into future value amount.

Therefore, we can say that the present value of a sum of money to be received at a future date is determined by discounting the future value at the interest rate that the money could earn over the period. This process is known as Discounting.

The present value interest rate or the future value interest rate is known as the discount rate. This discount rate is the rate with which the present value or the future value is traded off. A higher discount rate will result in a lower value for the amount in the future. This rate also represents the opportunity cost as it captures the returns that an individual would have made on the next best opportunity.

Since finding present value is simply the reverse of finding Future Value (FV), the formula for Future Value (FV) can be readily transformed into a Present Value formula. Therefore the  $P_0$ , the Present Value becomes:-

$$P_0 = \frac{FV_n}{(1+i)^n} \text{ OR } P_0 = FV_n (1+i)^{-n}$$

Where,  $FV_n$  = Future value n years hence  
 $i$  = Rate of interest per annum  
 $n$  = Number of years for which discounting is done.

As mentioned earlier, computation of P may be simple if we make use of either the calculator or the Present Value table showing values of  $(1+i)^{-n}$  for various time periods/per annum interest rates. For positive i, the factor  $(1+i)^{-n}$  is always less than 1, indicating thereby, future amount has smaller present value.

**Illustration 8:** *What is the present value of ₹ 1 to be received after 2 years compounded annually at 10%?*

**Solution**

Here  $FV_n = 1$ ,  $i = 0.1$

Required Present Value =  $FV_n (1+i)^{-n}$

$$= \frac{FV_n}{(1+i)^n} = \frac{1}{(1.1)^2} = \frac{1}{1.21} = 0.8264 = ₹ 0.83$$

Thus, ₹ 0.83 shall grow to ₹ 1 after 2 years at 10% compounded annually.

**Illustration 9:** Find the present value of ₹ 10,000 to be required after 5 years if the interest rate be 9 per cent. Given that  $(1.09)^5 = 1.5386$

**Solution**

Here,  $i = 0.09$ ,  $n = 5$ ,  $FV_n = 10,000$

Required Present value =  $FV_n (1+i)^{-n}$

$$= 10,000 (1.09)^{-5} = 10,000 \times 0.65 = ₹ 6,500.$$

$$\left[ (1.09)^{-5} = \frac{1}{(1.09)^5} = 0.65 \right]$$

**Illustration 10:** Find out the present value of ₹ 2,000 received after 10 years if discount rate is 8%.

**Solution**

Present value of an amount =  $FV_n \left( \frac{1}{1+i} \right)^n$

Now,  $i = 8\%$

$n = 10$  years

Present value of an amount = ₹ 2,000  $\left( \frac{1}{1+0.08} \right)^{10}$

$$= ₹ 2,000 (0.463) = ₹ 926$$

**Illustration 11:** What is the present value of ₹ 50,000 to be received after 10 years at 10 per cent compounded annually?

**Solution**

Here  $n = 10$ ,  $i = 0.1$

$P = FV_n (1+i)^{-n}$

$$= 50,000 (1.1)^{-10} = 50,000 \times 0.385543 = ₹ 19,277.15$$

## 2.7 Annuity

An annuity is a stream of regular periodic payment made or received for a specified period of time. In an ordinary annuity, payments or receipts occur at the end of each period.

**2.7.1 Future Value of an Annuity:** Expressed algebraically,  $FVA_n$  is defined as future (compound) value of an annuity,  $R$  the periodic receipt (or payment), and  $n$  the length of the annuity, the formula for  $FVA_n$  is:-

$$FVA_n = R(1+i)^{n-1} + R(1+i)^{n-2} + \dots + R(1+i)^1 + R(1+i)^0$$

As we can see,  $FVA_n$  is simply equal to the periodic receipt ( $R$ ) times the "sum of the future value interest factors at  $i$  percent for time periods 0 to  $n-1$ ."

As a shortcut, If  $R$  be the periodic payments, the amount  $FVA_n$  of the annuity is given by:

$$FVA_n = R \frac{(1+i)^n - 1}{i}$$

OR  $FVA_n = R (FVIFA_{i,n})$

Where  $FVIFA_{i,n}$  stands for the future interest factor of an annuity at  $i\%$  for  $n$  periods.

Table for  $FVA_n$  at different rates of interest may be used conveniently, if available, to work out problems. The value of expression  $\frac{(1+i)^n - 1}{i}$  or  $FVIFA_{i,n}$  can easily be found through financial tables.

**Illustration 12:** Find the amount of an annuity if payment of ₹ 500 is made annually for 7 years at interest rate of 14% compounded annually.

**Solution**

Here  $R = 500$ ,  $n = 7$ ,  $i = 0.14$

$$FVA = ₹ 500 \times FVIFA(7, 0.14) = 500 \times 10.7304915 = ₹ 5,365.25$$

**Illustration 13 :** A person is required to pay four equal annual payments of ₹ 5,000 each in his deposit account that pays 8% interest per year. Find out the future value of annuity at the end of 4 years.

**Solution**

$$\begin{aligned} FVA &= R \left( \frac{(1+i)^n - 1}{i} \right) \\ &= ₹ 5,000 (4.507) = ₹ 22,535 \end{aligned}$$

**Illustration 14:** ₹ 200 is invested at the end of each month in an account paying interest 6% per year compounded monthly. What is the amount of this annuity after 10th payment? Given

that  $(1.005)^{10} = 1.0511$

**Solution**

We have  $A(n,i) = \frac{(1+i)^n - 1}{i}$ ,  $i$  being the interest rate (in decimal) per payment period over  $n$  payment period.

Here,  $i = 0.06/12 = 0.005$ ,  $n = 10$ .

Required amount is given by  $A = P.A(10, 0.005)$

$= 200 \times 10.22 = ₹ 2,044$ .

**2.7.2 Present Value of an Annuity:** Sometimes instead of a single cash flow the cash flows of the same amount is received for a number of years. The present value of an annuity may be expressed as follows:

$$\begin{aligned} PVA_n &= \frac{R}{(1+i)^1} + \frac{R}{(1+i)^2} + \dots + \frac{R}{(1+i)^{n-1}} + \frac{R}{(1+i)^n} \\ &= R \left( \frac{1}{(1+i)^1} + \frac{1}{(1+i)^2} + \dots + \frac{1}{(1+i)^{n-1}} + \frac{1}{(1+i)^n} \right) \\ &= R (PVIF_{i,1} + PVIF_{i,2} + PVIF_{i,3} + \dots + PVIF_{i,n}) \\ &= R (PVIF_{i,n}) \end{aligned}$$

Where,

- $PVA_n$  = Present value of annuity which has duration of  $n$  years
- $R$  = Constant periodic flow
- $i$  = Discount rate and,
- $(PVIF_{i,n})$  = Present value interest factor of an (ordinary) annuity at  $i$  percent for  $n$  periods.

**Illustration 15:** Find out the present value of a 4 year annuity of ₹ 20,000 discounted at 10 per cent.

**Solution**

$PVA$  = Amount of annuity  $\times$  Present value ( $r, n$ )

Now,  $i$  = 10%

$N$  = 4 years

$$PVA = ₹ 20,000 \left[ \frac{(1+0.1)^4 - 1}{0.1(1+0.1)^4} \right] = ₹ 20,000 \times 3.17 = ₹ 63,400$$

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**Illustration 16:** Y bought a TV costing ₹ 13,000 by making a down payment of ₹ 3,000 and agreeing to make equal annual payment for 4 years. How much would be each payment if the interest on unpaid amount be 14% compounded annually?

### Solution

In the present case, present value of the unpaid amount was  $(13,000 - 3,000) = ₹ 10,000$ . The periodic payment, R may be found from

$$R = \frac{PVA}{PVIF(i, n)} = \frac{10,000}{PVIF(0.14, 4)} = \frac{10,000}{2.914} = ₹ 3,431.71$$

**Illustration 17:** Z plans to receive an annuity of ₹ 5,000 semi-annually for 10 years after he retires in 18 years. Money is worth 9% compounded semi-annually.

- How much amount is required to finance the annuity?
- What amount of single deposit made now would provide the funds for the annuity?
- How much will Mr. Z receive from the annuity?

### Solution

- (a) Let us first find the required present value for the 10 years annuity by using

$$\begin{aligned} PVA &= R[PVIF(i, n)] \\ &= 5,000 [PVIF(4.5\%, 20)] \\ &= 5,000 \times 13.00793654 = ₹ 65,039.68 \end{aligned}$$

$$\begin{aligned} \text{Since, } PVIF(4.5\%, 20) &= \frac{(1 + 4.5\%)^{20} - 1}{.045(1 + 4.5\%)^{20}} \\ &= \frac{2.41171402 - 1}{0.10852713} = 13.00793654 \end{aligned}$$

- (b) We require the amount of single deposit that matures to ₹ 65,039.68 in 18 years at 9% compounded semi-annually. We use the following formula:-

$$P_0 = FV_n(1 + i)^{-n}$$

$$\text{Where } FV_n = 65,039.68, n = 18 \times 2 = 36, i = \frac{9}{2} = 4 \frac{1}{2}\%, P_0 = ?$$

$$\text{Thus, } P_0 = 65,039.68 \left(1 + 4 \frac{1}{2}\%\right)^{-36} = 65,039.68 \times 0.20502817 = ₹ 13,334.97$$

- (c) Required Amount = ₹ 5,000 x 20 = ₹ 1,00,000

**Illustration 18:** Determine the present value of ₹ 700 each paid at the end of each of the next six years. Assume an 8 per cent of interest.

**Solution**

As the present value of an annuity of ₹ 700 has to be computed. The present value factor of an annuity of ₹ 1 at 8 per cent for 6 years is 4.623. Therefore, the present value of an annuity of ₹ 700 will be:  $4.623 \times ₹ 700 = ₹ 3,236.10$

**2.8 Loan Amortisation & Capital Recovery**

If we receive some amount from the lender at a given rate of interest for a given period then we can calculate the amount of instalment (constant periodic flow) to pay to the lender as an instalment:

$$R = \frac{PVA_n}{PVIFA_{in}}$$

Reciprocal of  $PVIFA_{in}$  is also known as capital recovery factor (CRF).

**Example:** Suppose you have borrowed a 3 year loan of ₹1,00,000 at 9 per cent from your employer to buy a motorcycle. If your employer requires three equal end-of-year repayments, then the annual instalment will be:

$$R = \frac{PVA_n}{PVIFA_{in}} \text{ or, } R = \frac{₹1,00,000}{PVIFA_{in}}$$

$$₹ 1,00,000 = R \times PVIFA_{(0.09, 3 \text{ years})}$$

$$₹ 1,00,000 = R \times 2.531 \text{ (from the } PVIFA(i,n) \text{ table)}$$

$$R = \frac{₹1,00,000}{2.531} = ₹ 39,510$$

By paying ₹ 39,510 each year for three years, you shall completely pay-off your loan with 9 per cent interest.

This can be observed from the loan-amortisation schedule given in Table

End of year	Payment	Interest	Principle Repayment	Outstanding Balance
0	--	--	--	1,00,000
1	39,510	9,000	30,510	69,490
2	39,510	6,254	33,256	3,6234
3	39,510	3,261	36,249	0

**2.9 Perpetuity**

Perpetuity is an annuity in which the periodic payments or receipts begin on a fixed date and continue indefinitely or perpetually. Fixed coupon payments on permanently invested (irredeemable) sums of money are prime examples of perpetuities.

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The formula for evaluating perpetuity is relatively straight forward. Two points which are important to understand in this regard are:

- The value of the perpetuity is finite because receipts that are anticipated far in the future have extremely low present value (today's value of the future cash flows).
- Additionally, because the principal is never repaid, there is no present value for the principal.

Therefore the price of perpetuity is simply the coupon amount over the appropriate discount rate or yield.

**2.9.1 Calculation of Multi Period Perpetuity:** The formula for determining the present value of multi-period perpetuity is as follows:

$$PVA_{\infty} = \frac{R}{(1+i)^1} + \frac{R}{(1+i)^2} + \frac{R}{(1+i)^3} + \dots + \frac{R}{(1+i)^{\infty}} = \sum_{n=1}^{\infty} \frac{R}{(1+i)^n} = \frac{R}{i}$$

Where:

R = the payment or receipt each period

i = the interest rate per payment or receipt period

**Illustration 19:** Ramesh wants to retire and receive ₹ 3,000 a month. He wants to pass this monthly payment to future generations after his death. He can earn an interest of 8% compounded annually. How much will he need to set aside to achieve his perpetuity goal?

**Solution** R = ₹ 3,000

$$i = 0.08/12 \text{ or } 0.00667$$

Substituting these values in the above formula, we get

$$\begin{aligned} PVA &= \frac{\text{₹ } 3,000}{0.00667} \\ &= \text{₹ } 4,49,775 \end{aligned}$$

If he wanted the payments to start today, he must increase the size of the funds to handle the first payment. This is achieved by depositing ₹ 4,52,775 (PV of normal perpetuity + perpetuity received in the beginning = 4,49,775 + 3,000) which provides the immediate payment of ₹ 3,000 and leaves ₹ 4,49,775 in the fund to provide the future ₹ 3,000 payments.

**2.9.2 Calculation of Growing Perpetuity:** A stream of cash flows that grows at a constant rate forever is known as growing perpetuity.

The formula for determining the present value of growing perpetuity is as follows:

$$PVA = \frac{R}{(1+i)^1} + \frac{R(1+g)}{(1+i)^2} + \frac{R(1+g)^2}{(1+i)^3} + \dots + \frac{R(1+g)^{\infty}}{(1+i)^{\infty}}$$

$$\sum_{n=1}^{\infty} \frac{R(1+g)^{n-1}}{(1+i)^n} = \frac{R}{i-g}$$

**Illustration 20:** Assuming that the discount rate is 7% per annum, how much would you pay to receive ₹ 50, growing at 5%, annually, forever?

**Solution**

$$PVA = \frac{R}{i-g} = \frac{50}{0.07 - 0.05} = 2,500$$

## 2.10 Sinking Fund

It is the fund created for a specified purpose by way of sequence of periodic payments over a time period at a specified interest rate.

Size of the sinking fund deposit is computed from  $FVA = R[FVIFA(i,n)]$ , where FVA is the amount to be saved, R, the periodic payment, n, the payment period.

**Illustration 21:** How much amount is required to be invested every year so as to accumulate ₹ 3,00,000 at the end of 10 years if the interest is compounded annually at 10%?

**Solution**

$$\text{Here, } FVA = 3,00,000 \quad n = 10 \quad i = 0.1$$

$$\begin{aligned} \text{Since, } FVA &= R[FVIFA(i,n)] \\ 3,00,000 &= R[FVIFA(0.10,10)] \\ &= R * 6.1146 \end{aligned}$$

$$\text{Therefore, } R = \frac{3,00,000}{6.1146} = 18,823.62 = R = ₹ 18,823.62$$

**Illustration 22:** ABCL Company has issued debentures of ₹ 50 lakhs to be repaid after 7 years. How much should the company invest in a sinking fund earning 12 percent in order to be able to repay debentures?

**Solution**

$$\begin{aligned} A (CVFA_{r,t}) &= 50,00,000 \\ A (CVFA_{0.12,7}) &= 50,00,000 \\ A &= \frac{50,00,000}{(CVFA_{0.12,7})} \\ A &= \frac{50,00,000}{10.089} = ₹ 4.96 \text{ lakhs.} \end{aligned}$$

**Illustration 23:** Bank of Delhi pays 8 per cent interest, compounded quarterly, on its money market account. The managers of Bank of Gurgaon want its money market account to equal Bank of Delhi's effective annual rate, but interest is to be compounded on monthly basis. What nominal, or quoted, or APR rate must Bank of Gurgaon set?

**Solution**

Bank of Delhi's effective annual rate is 8.24 per cent:

$$\text{Effective annual rate} = \left(1 + \frac{0.08}{4}\right)^4 - 1.0 = (1.02)^4 - 1 = 1.0824 - 1 = 0.0824 = 8.24\%.$$

Now, Bank of Gurgaon must have the same effective annual rate:

$$\left(1 + \frac{i}{12}\right)^{12} - 1.0 = 0.0824$$

$$\left(1 + \frac{i}{12}\right)^{12} = 1.0824$$

$$1 + \frac{i}{12} = (1.0824)^{1/12}$$

$$1 + \frac{i}{12} = 1.00662$$

$$\frac{i}{12} = 0.00662$$

$$i = 0.07944 = 7.94\%.$$

Thus, the two banks have different quoted rates – Bank of Delhi's quoted rate is 8%, while Bank of Gurgaon's quoted rate is 7.94%; however, both banks have the same effective annual rate of 8.24%. The difference in their quoted rates is due to the difference in compounding frequency.

### SUMMARY

- Money has time value.
- A rupee today is more valuable than a rupee a year hence.
- We use rate of interest to express the time value of money.
- Simple Interest may be defined as Interest that is calculated as a simple percentage of the original principal amount.  
Formula :  $SI = P_0 (i)(n)$
- Compound interest is calculated on total of previously earned interest and the Original Principal.

- The Present Value of a sum of money to be received at a future date is determined by discounting the future value at the interest rate that the money could earn over the period

$$\text{Formula: } P_0 = \frac{FV_n}{(1 + i)^n} \text{ OR } P_0 = FV_n (1 + i)^{-n}$$

- Future Value is the value at some future time of a present amount of money, or a series of payments, evaluated at a given interest rate.

$$\text{Formula: } FV_n = P_0 + SI = P_0 + P_0(i)(n) \text{ or}$$

$$FV_n = P_0 \left( 1 + \frac{r}{k} \right)^n$$

- An annuity is a series of equal payments or receipts occurring over a specified number of periods.

a. Present value of an ordinary annuity – cash flows occur at the end of each period, and present value is calculated as of one period before the first cash flow.

b. Present value of an annuity due – cash flows occur at the beginning of each period, and present value is calculated as of the first cash flow.

$$\text{Formula: } PVA_n = R (PVIFA_{i,n})$$

c. Future value of an ordinary annuity – cash flows occur at the end of each period, and future value is calculated as of the last cash flow.

d. Future value of an annuity due – cash flows occur at the beginning of each period, and future value is calculated as of one period after the last cash flow.

$$\text{Formula: } FVA_n = R (FVIFA_{i,n})$$