

Benford Distribution - An Effective Audit Tool



The Benford analysis is a powerful and relatively simple tool for pointing out false or wrong accounts and can be effectively used in auditing. In all phases of audit the application of this powerful tool enables the audit executive to discharge his duties effectively. The careful application of Benford analysis leads to useful conclusions. This article demonstrates an effective method of locating mistakes in a very large-scale auditable data-set by applying statistical test on Benford distribution. The identification of abnormally behaved data and making in-depth analysis of those data will always lead to identify and prevent fraudulent transactions. The tool explained in this article helps the audit executive to identify objectionable data in order to perform their task more effectively, efficiently and economically within a short span of time.



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Audit Control

Control is one of the important managerial functions, which helps to check errors and take corrective action so that deviation from standards are minimised and stated goals of the organisation are achieved in a desired manner. Audit Control, which is one of organisational controls, consists of setting audit criteria, evaluating financial records and suggesting

corrective action in order to improve the performance of the organisation.

The executive, who is exercising audit control has necessarily to conform, whether the activities of an organisation are carried out in accordance with the plan, which has been adopted, the orders, which have been given, and the principles, which have been laid down, in order to identify deviations

and also recommend corrective measures for rectification, which facilitate prevention and recurrence of mistakes.

The audit executive adopts various analytical procedures with the objective to reduce the audit risk to the minimum level for efficiently discharging this primary duty entrusted with him. The application of various tools and techniques from audit planning until reporting will enable him to derive scientific and useful conclusions. The audit executive has to estimate the probable mistakes most likely to exist within the organisation at the time of audit planning itself by making an in-depth analysis of inherent risk, control risk and other risks existing within the organisation. In this audit-planning stage, the auditor has to adopt various techniques such as Design analysis, Parato analysis, Zif's analysis, Benford analysis and other analysis in order to estimate the efficiency- economy- effectiveness of the various functions of the entity.

Many researchers analysed the indifferent phenomenon of Benford law and offered various explanations. In 1976, Raimi has stated in his article that this phenomenon is the result of 'the way we write numbers' and further stated that Benford law reflects a profound harmonic truth of nature. Boyle (1994) proved that Benford distribution is 'scale invariant'. He found that the first digit taken from a sample is independent of the measuring units used. If the entire data multiplied by a constant, the resultant data also obeys Benford's Law. 

After appropriate audit planning, the audit executive has to prescribe audit criteria by conducting preliminary survey, interview as well as making an in-depth analysis of previous case studies, etc. After fixing the audit criteria and identification of critical areas of auditing, he has to confirm the vertical, horizontal and dimensional consistency of the accounts. The ratio analysis, trend analysis, factor analysis, correlation analysis, as well as Benford analysis will enable him to identify the abnormally behaved variables in order to have thorough check for the identification of the reasons of the abnormal behaviour. He has to vouch for all the expenditure and income for the period of review. When complete checking is not possible, the audit executive has to adopt appropriate statistical techniques in order to derive scientific conclusions.

The distribution of leftmost leading digits in data of an anomalous nature will conform to a formula of logarithmic intervals known as the Benford distribution. The Benford analysis is a powerful and relatively simple tool for pointing out false or wrong accounts and can be effectively used in auditing. In all phases of audit the application of this powerful tool enables the audit executive to discharge his duties effectively. This simple and efficient tool, like Parato analysis, assists the audit executives to confirm the correct recording of incomes, expenditure, assets, and liabilities.

The careful application of Benford analysis leads to useful conclusions. The primary intention of this article is to demonstrate, how and where the Benford's principles could be effectively used in auditing.

History of Benford Law

The principles of Benford law were first published in *American Journal of Mathematics* during 1881. Simon

It is true that not all data sets followed Benford's law. Assigned numbers, such as PAN card numbers prescribed by the Income Tax Department, PIN codes or Bank account numbers will not conform to Benford's law, as they are pre assigned. As an example, in the pre-determined refund given to the customers, the Benford law may not be valid. If the account balance has built-in maximum or minimum balance, the Benford test will not lead good results. 

Newcomb published an article in the said American journal and concluded that more numbers exist, which begin with number one than with other numbers.

During 1938, Dr. Frank Benford, a physicist of the General Electric Company had rediscovered this technique. He did a detailed mathematical analysis of more than 20,000 sets of numbers such as stock quotations, tournament tennis scores, the electricity bills, etc. These unrelated sets of Anomalous Numbers followed the same first-digit probability pattern and in all cases the number one turned up as the first digit in about 30% of the time, more often than any other number.

By this practical experience, he derived a formula and concluded that in a given string of at least four numbers sampled from one or more of any data, the chance that the first digit 1 is 30.1%, but not 11.11% (i.e. not 1/9) as normally expected, but it is 30.1%. The chance that the first digit 2 is in the string is only 17.6%, and the probabilities that successive numbers will be the first digit decline smoothly up to 9,

which has only a 4.6% chance, as detailed below.

Digit	1	2	3	4	5	6	7	8	9
Probability of obtaining as 1 st digit (%)	30.1%	17.6%	12.5%	9.7%	7.9%	6.7%	5.8%	5.1%	4.6%

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An interesting case was noticed in the literature of Benford analysis. In the State of Arizona, the manager of the State treasury was charged that he had diverted funds to bogus vendors. Benford analysis test was applied in the said case and it was found that the digits 7, 8 and 9 occur more than 90%, which is much contrary to the Benford's law. A detail analysis was made and it was found that most of the amount were just below 1,00,000/- for avoiding the check of higher authorities. It was also found that none of the amount was duplicated and there were no round numbers. However, the charged official repeated some digits and digit combinations. The Benford test revealed the fraudulent payment

and accordingly the official was punished.

The Benford analysis may reveal the peculiarity of the transactions. In a particular case, while analysing the accounts payable data of a company it was found that there was large first two-digit spike in excess of the expected Benford frequency. While analysing the reasons, it was found that the company has directed to submit vouchers if the amount exceeds the prescribed amount. The employees for avoiding the submission of vouchers claimed lesser amount for which no vouchers need to be submitted. Similarly, the payments might be split into 2 or 3 parts for avoiding approval of higher authorities; such spikes could be easily identified by the Benford analysis.

Applications of Benford's Law in Auditing

The audit executives are necessarily to apply various digital analysis and analytical procedures for effective performance of their duties. As the SIA-6¹ states that the auditor has to employ analytical procedures, the audit executives have to discover and employ various analytical procedures for effectively discharging their responsibilities. Benford analysis is one of the vital tools and could be applied in almost all phases of auditing i.e. from audit planning stage to reporting stage.

The correctness of information received from the auditee entity in audit planning stage could be checked by applying Benford's analysis. It is also possible to get clue for duplicate payments, missing data by identifying

duplicate purchase orders or invoice numbers in audit execution stage by adopting this technique.

As majority of the accounting data follow Benford distribution, the audit executives could use this technique. While applying Benford analysis, it is necessary to consider the entire data. Samples taken from the data will not yield good results. As an example, if the audit executive wants to check the accounts receivable or accounts payable, the entire transactions have to be analysed. The audit executive cannot ascertain exact position, on the samples taken from the accounts payable or accounts receivable.

The account size i.e. number of transactions in an account has to be considered before applying Benford analysis. The large number of data or items will always lead to good result. Therefore, the audit executive has to apply the Benford test on the transactions of the entire year.

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¹ Standard On Internal Audit (Sia) 6 - Analytical Procedures*

such accounts will not satisfy the Benford test.

Similarly, for the transactions such as thefts, kickbacks, contract rigging, etc, the Benford analysis could not be effectively used. By removing the abnormal data stated above, the remaining data will confirm Benford test, if accounts reflects correct position. By adopting the appropriate strategies, it is always possible to overcome the limitations. As an example, this tool will indicate that there may be fraud or in built mechanism within the accounts. This could be easily identified by the auditor and in case if he identifies that there is no scope for built-in mechanism, there may be possibility of mistakes/frauds/irregularities in such areas, he could take up a detailed audit.

However, Benford analysis could be used as a primary tool to be added to the audit executive's weapon store. Audit executive has to use additional tests, which include a personal awareness of the examined observations of assets, as well as awareness of corporate culture, etc to derive useful conclusions.

Steps Involved in Applying Benford Test

Step 1 - Check whether the accounting balances confirm Benford distribution

The first step is to confirm whether the type of accounts satisfy the Benford's distribution. It is necessary to ensure whether the accounts consist of pre-assigned balances, or has inbuilt maximum or minimum balances or the transactions are in the nature of contract rigging, kickbacks or of theft types. As the Benford distribution is not valid in such cases, Benford test on these transactions will not yield any required results. The accounting data relating to accounts payable, accounts receivable, refunds, sales, purchases, inventory cost and

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their unit prizes, insurance claims, employee expenses, fixed assets and liabilities, etc. normally follow Benford distribution and hence Benford law could be effectively used in such cases.

Step 2- Finding frequencies of first digit from the Account Balances

The next step is to extract the first digit from the account balances of the data under scrutiny. After

identifying the first digit in the accounting balances, the next step is to find the frequency of the first digits identified from the accounting balances.

Step 3- Comparison with predetermined frequencies

The Benford law predetermines the frequencies of first digit as mentioned in the Table 1. The predetermined first digit frequencies have to be compared with actual frequencies as found in the Step 2 and the difference between these could be found. The audit executive uses his discretion to allow the difference. Normally the difference of 5% i.e. -2.5% to +2.5% is admissible. However, according to the circumstances of the case and discretion of the auditors, it could be increased or decreased by the audit executives. The digit, which does not confirm to the prescribed admissible difference, has to be identified.

Step 4- Analysis of the abnormal digits

There may be many reasons for the abnormal behaviour of the digits



identified. The auditor has to list all the balances, which behaved abnormally and make detailed audit checks on the balances identified. If case specific peculiarities of the transactions are to be found or the practice adopted is to be identified, such as consolidation of refunds and issue of one cheque instead of several cheques, such account balances are to be removed. After removing, the peculiar balances from the purview of Benford analysis, and making necessary corrections after applying required audit checks, the corrected balance account should be re-checked by adopting Benford analysis. The next stage is to adopt the appropriate statistical test to confirm whether frequencies of digit are consistent with Benford's distribution.

Step 5 - Benford's Distribution Analysis

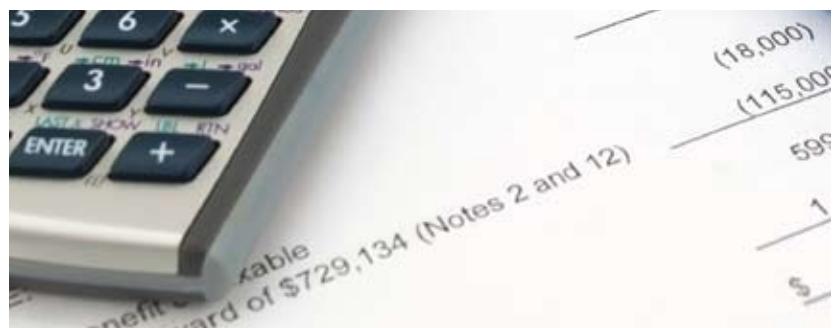
The auditor may use appropriate tests to confirm the whether digits in the data set under consideration are distributed in accordance with Benford law. The audit executive can use appropriate statistical test for deriving useful conclusions.

Step 6- Identification of the auditable area - Adoption of 'Divide and Rule' Policy

After exercising the procedures, mentioned in the above steps and removal of abnormal terms, there may be chance that the data under consideration may not conform to the Benford test. In such cases, the entire data set could be divided into two parts and Benford test could be applied to both parts separately.

There may be chances that either half of the data may not satisfy the Benford test or either of the one half may not satisfy the analysis. If any one-half of the data satisfies, such half could be left.

The other half of the data, which does not satisfy Benford analysis, may again be broken into further



two parts and the Benford analysis has to be done on both the parts in order to confirm whether they satisfy the Benford analysis. If any half of the broken data does satisfy statistical tests, that portion could be ignored and the remaining half has to be concentrated upon.

This exercise has to be continued until the data in half falls below minimum of 100 or any lower limit prescribed by the audit executive at his discretion. By this exercise, the broken data which satisfy the Benford analysis could be left out and a detailed audit analysis has to be undertaken on the remaining data, which does not satisfy the Benford analysis. If the entire data does not satisfy the Benford analysis there is no alternative except to audit all the

transactions of the data under consideration in detail. By adopting this technique, the workload of the audit executive is reduced to a minimum level and the audit executive could concentrate on few records instead of entire records.

Step 7 – Refining the tests

There is a possibility that the Benford test on first digit alone may not reveal correct results. The analysis could be extended to the two digits in order to identify the spikes in the frequency. The same procedure as explained above could be adopted. The first-two-digits test could be fine-tuned to a first-three-digits test also. In all cases, the sample size should be manageable and could not be excessive and extensive.

Conclusion

There is no effective audit tool available as on date for identification of all types of mistakes/ frauds/ irregularities. This new simple and easy analysing audit tool helps the auditor to identify the abnormal transactions and assists him to perform his task more effectively, efficiently and economically within the short span of time. This new audit tool will identify the areas where the actual mistakes/ frauds/irregularities might have occurred. The new innovative idea of breaking the data and applying the Benford analysis will no doubt reduce auditor's precious time and also improve the efficiency of the auditor in identification of the fault areas. ■

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