14

Program Evaluation and Review Technique

LEARNING OBJECTIVES

After studying this unit you will be able to :

- Work out how network analysis helps managers to plan when to start various tasks to monitor actual progress and to know when control action is needed to prevent delay in completion of the project
- Draw diagram and chart representing inter-relationships between various elements of the project and finally establishing critical path
- Identify dummy activities
- Identify earliest event times and latest event times for each event
- Identify total float, free float and independent float
- Crash times, costs and consumption of resources
- Help in decision making about project times and resource allocation
- Calculate the probability of completing the network in a given time when multiple time estimates of activity duration are available

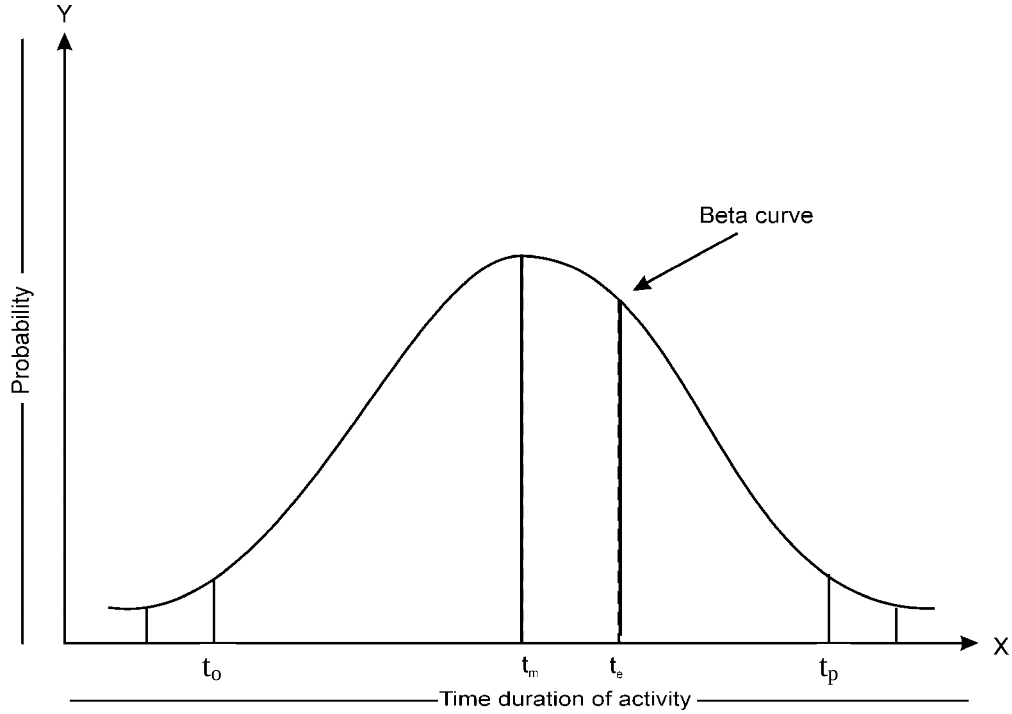
14.1 Introduction

Hitherto our emphasis has been on CPM which is applicable where activity durations can be estimated from experience, past historical records and work study techniques fairly precisely. *CPM is incapable of handling uncertainty in timing* which is a rule rather than an expectation for innovational projects such as introducing a new product or oil exploration project. PERT (Program Evaluation and Review Technique) is more relevant for handing such projects which have a great deal of uncertainty associated with the activity durations. To take these uncertainty into account, three kinds of times estimates are generally obtained. These are:

(a) *The Optimistic Times Estimate:* This is the estimate of the shortest possible time in which an activity can be completed under ideal conditions. For this estimate, no provision for delays or setbacks is made. We shall denote this estimate by t_o.

(b) *The Pessimistic Time Estimate:* This is the maximum possible time which an activity could take to accomplish the job. If everything went wrong and abnormal situations prevailed, this would be the time estimate. It is denoted by t_p.

(c) *The Most Likely Time Estimate :* This is a time estimate of an activity which lies between the optimistic and the pessimistic time estimates. It assumes that things go in a normal way with few setbacks. It is represents by t_m. Statistically, it is the model value if duration of the activity.



t_o = optimistic time
t_m = most likely time
t_e = expected time
t_p = pessimistic time

Figure 1

Estimate of Probability: Due to variability in the activity duration, the total project may not be completed exactly in time. Thus, it is necessary to calculate the probability of actual meeting the schedule time of the project as well as activities.

Probability of completing the project by schedule time (T_s) is given by

$$Z = \frac{T_s - T_e}{\sigma_e}$$

Te represents the duration on the critical path. T_e can be calculated by adding the expected time of each activity lying on the critical path. σ_e represents standard deviation of the critical path. Variance of the critical path can be getting by adding variances of critical activities. σ_e is the square root of variance of the critical path.

Beta distribution is a assumed for these "guess estimates" and PERT analysts have found that beta-distribution curve happens to give fairly satisfactory results for most of the activities. For a distribution of this type, the *standard deviation* is approximately one sixth of the range, i.e.

$$S_t = \frac{t_p - t_o}{6}$$

The variance, therefore; is

$$S_t^2 = \left(\frac{t_p - t_o}{6}\right)^2$$

Expected time: The expected time (t_e) is the average time taken for the completion of the job. By using beta-distribution, the expected time can be obtained by following formula.

$$t_e = \frac{t_o + 4t_m + t_p}{6}$$

Illustration 1

If the critical path of a project is 20 months along with a standard duration of 4 months, what is the probability that the project will be completed within:

(a) 20 months (b) 18 months (c) 24 months?

Solution

(a) $Z = \frac{20 - 20}{4} = 0$; Probability = 0.50

(b) $Z = \frac{18 - 20}{4} = -0.50$; Probability = 0.31

(c) $Z = \frac{24 - 20}{4} = 1$; Probability = 0.84

Illustration 2

PERT calculations yield a project length of 60 weeks with variance of 9. Within how many weeks would you expect the project to be completed with probability of 0.99?

Solution

$T_e = 60$ S.D. $= \sqrt{9} = 3$

$$Z = \frac{T_s - T_e}{\sigma} = \frac{T_s - 60}{3} = 2.3$$

$60 + 3 \times 2.3 = 67$ weeks

Note: Prob. 0.99 means Z = 2.3

Illustration 3

A small project is composed of 7 activities whose time estimates are listed in the table below. Activities are identified by their beginning (i) and ending (j) node numbers.

(a) Draw the project network and identify all the paths through it.

(b) *Find the expected duration and variance for each activity. What is the expected project length?*

(c) *Calculate the variance and standard deviation of project length. What is the probability that the project will be completed?*

(i) *At least 3 weeks earlier than expected?*

(ii) *No more than 3 weeks later than expected?*

(d) *If the project due date is 18 weeks, what is the probability of not meeting the due date?*

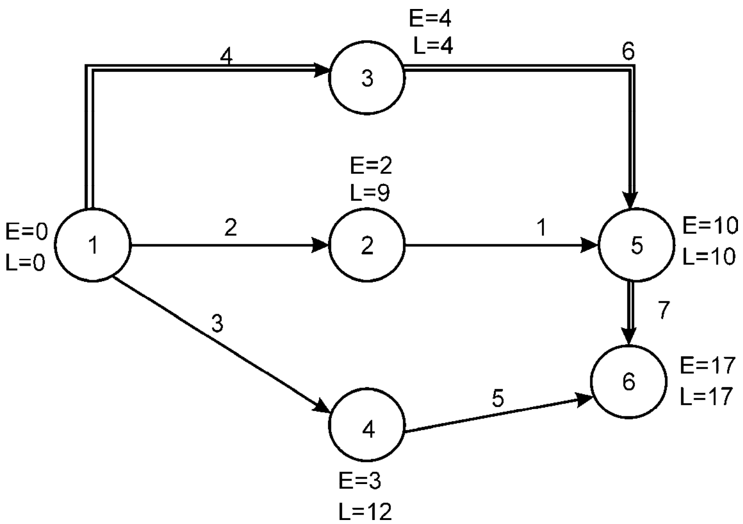
(e) *What due date has about 90% chance of being met?*

	Activity Estimated duration in Weeks		
i–j	*Optimistic*	*Most Likely*	*Pessimistic*
1–2	*1*	*1*	*7*
1–3	*1*	*4*	*7*
1–4	*2*	*2*	*8*
2–5	*1*	*1*	*1*
3–5	*2*	*5*	*14*
4–6	*2*	*5*	*8*
5–6	*3*	*6*	*15*

Solution

(a) The network is drawn in Fig. 2. The various paths are as follows.

1–2–5–6, 1–3–5–6, 1–4–6



Critical Path 1–3–5–6

Figure 2

(b) Expected duration and variances for various activities are computed below.

	T_e		V_t	
1–2;	$\frac{1+1\times 4+7}{6}$	= 2;	$\left[\frac{7-1}{6}\right]^2$	$=1$
1–3;	$\frac{1+16+7}{6}$	= 4;	$\left[\frac{7-1}{6}\right]^2$	$=1$
1–4;	$\frac{2+8+8}{6}$	=3;	$\left[\frac{8-2}{6}\right]^2$	$=1$
2–5;	$\frac{1+4+1}{6}$	=1;	$\left[\frac{1-1}{6}\right]^2$	$=0$
3–5;	$\frac{2+20+14}{6}$	=6;	$\left[\frac{14-2}{6}\right]^2$	$=4$
4–6;	$\frac{2+20+8}{6}$	=5;	$\left[\frac{8-2}{6}\right]^2$	$=1$
5–6;	$\frac{3+24+15}{6}$	=7;	$\left[\frac{15-3}{6}\right]^2$	$=4$

(c) Expected project Length = 17 weeks

Variance of the critical path 1–3–5–6 = 1+4+4 = 9

$\therefore$ Standard Deviation = $\sqrt{9}$ = 3

(i) $Z = \frac{14-17}{3} = -1$; Probability = 0.159

(ii) $Z = \frac{20-17}{3} = +1$; Probability = 0.841

(d) $T_e = 17$; $T_1 = 18$; $Z = \frac{18-17}{3} = 0.333.$

Therefore, probability of meeting the due date = 0.63.

And probability of not meeting the date = 0.37 (1–0.63)

(e) At 90% Probability, Z = 1.3 approximately.

$\therefore 1.3 = \frac{T_s - 17}{3}$ or T_S = 20.9 weeks approximately.

☞ Critical Path is highlighted through double line arrows.

Illustration 4

Find event variances in the network of previous illustration.

Solution

Event variance for both the T_E and T_L of each event are derived below. The computational procedure should be self evident. We shall put to use the variances of the various activities derive in part (b) of solution to example above.

Earliest Time, T_E (D=Duration)

Event	*Longest path leading to it*	**Variance*
1	Nil (D=0)	0
2	1–2 (D=2)	1
3	1–3 (D=4)	1
4	1–4 (D=3)	1
5	1–3, 3–5 (D = 4 + 6 = 10)	1 + 4 = 5
6	1–3, 3–5, 5–6 (D=17)	1 + 4 + 4 = 9

Suppose we wish to find the probability of reaching event 5 in 9 days. This can be computed as below:

$$Z = \frac{9-10}{\sqrt{5}} = \frac{(-)1}{\sqrt{5}} = \frac{-\sqrt{5}}{5} = \frac{(-)\ 2.236}{5} = -0.4472$$

Probability of reaching event 5 in 9 days is equal to 0.5 – .1723 = .3275. Likewise we can determine probabilities of reaching other events.

Longest Time T_L

Event	*Longest path from it to last event 6*	*Variance*
1	1–3, 3–5, 5–6	9
2	2–5, 5–6	4
3	3–5, 5–6	8
4	4–6	1
5	5–6	4
6	Nil	0

Illustration 5

Shown below is a PERT network and a related set of activity times:

i–j	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*	*L*
t_o	10	12	8	4	0	12	6	9	4	0	5	9
t_m	13	15	11	7	0	18	12	12	6	0	8	12
t_p	22	18	20	16	0	36	18	27	8	0	11	33

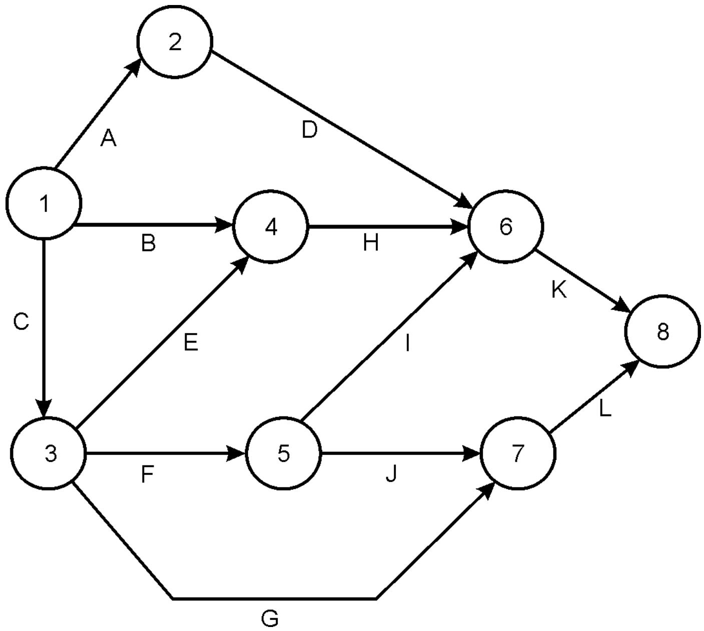


Figure 3

Required

(a) *Determine the expected completion time of each activtiy.*

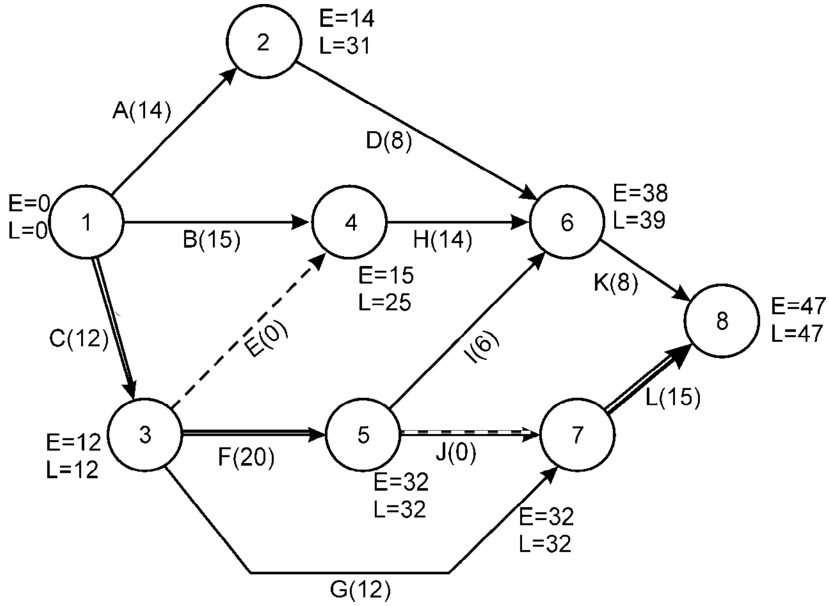
(b) *Determine the earliest expected completion time, the latest expected completion time and float of each activity.*

(c) *What is the total project completion time, and what are activities on the critical path?*

(d) *Determine S.D. of expected completion time for only those activities on the critical path.*

(e) *Determine the probability that the project will be completed within 41 weeks.*

Solution



1–3–5–7–8

Figure 4

$$T_e = \frac{t_p + 4t_m + t_0}{6}\ ;\ V_t = \left(\frac{t_p - t_0}{6}\right)^2$$

	Time estimates				Start		Finish		LST – EST		
i–j	t_o	$4t_m$	t_p	t_e	EST	LST	EFT	LFT	Float	V_t	S.D
1–2	10	52	22	14	0	17	14	31	17		
1–3	8	44	20	12	0	0	12	12	0	4	2
1–4	12	60	18	15	0	10	15	25	10		
2–6	4	28	16	8	14	31	22	39	17		
3–4	0	0	0	0	12	25	12	25	13		
3–5	12	72	36	20	12	12	32	32	0	16	4
3–7	6	48	18	12	12	20	24	32	8		
4–6	9	48	27	14	15	25	29	39	10		
5–6	4	24	8	6	32	33	38	39	1		
5–7	0	0	0	0	32	32	32	32	0	0	0
6–8	5	32	11	8	38	39	46	47	1		
7–8	9	48	33	15	32	32	47	47	0	16	4
									$\Sigma V_t = 36$		

Hence S.D. of the critical path = $\sqrt{36}$ = 6

Probability of completion critical path in 41 weeks is computed below:

$$Z = \frac{41 - 47}{6} = -1$$

$\therefore$ Probability = 0.159 (Answer).

14.2 Updating the Network

The progress of various activities in a project network is measured periodically. Normally, either most of the activities are ahead or behind the schedule. It is therefore, necessary to update or redraw the network periodically to know the exact position of completion of each activity of the project. The task of updating the network may be carried out once in a month. Sometimes the updating of the network may provide useful information to such an extent that it may demand the revision of even those very activities which have not started. Even the logic may also change i.e. some of the existing activities may have to be dropped and new activities may be added up. In brief the network should be amended accordingly in the light of new developments.

It is also not unlikely that the total physical quantum of work accomplished at a point of time may exceed what was planned but the progress against the critical path alone may be slower than the scheduled pace. To understand how the task of updating is carried out, consider the following example:

Illustration 6

After 15 days of working the following progress is noted for the network of an erection job.

(a) Activity 1-2, 1-3 and 1-4 completed as per original schedule.

(b) Activity 2-4 is in progress and will be completed in 3 more days.

(c) Activity 3-6 is in progress and will need 18 days more for completion.

(d) Activity 6-7 appears to present some problem and its new estimated time of completion is 12 days.

(e) Activity 6-8 can be completed in 5 days instead of originally planned for 7 days.

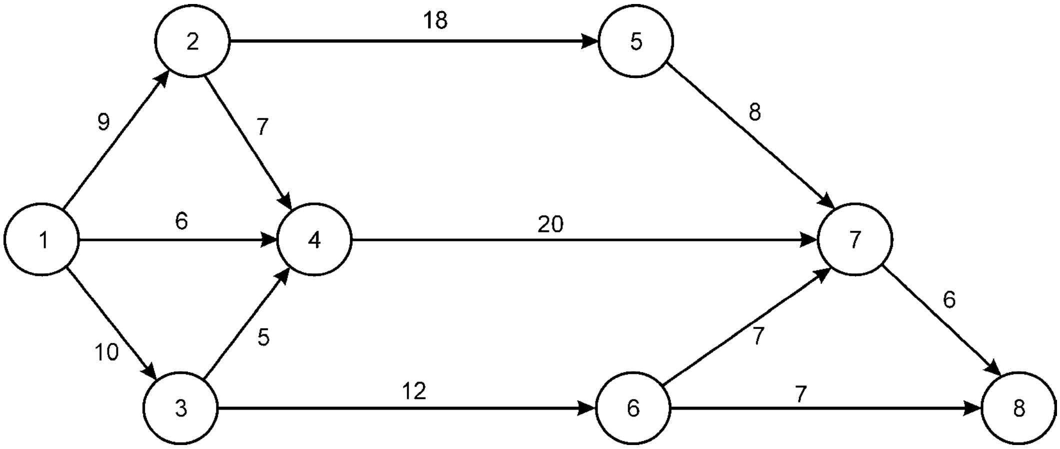


Figure 5

You are required to:

(i) *Update the above diagram after 15 days of the start of work based on the assumptions given above.*

(ii) *Write down the critical path with total project duration.*

Solution

Path	Duration
1–2–5–7–8	9+18+8+6 = 41
1–2–4–7–8	9+7+20+6 = 42
1–4–7–8	6+20+6 = 32
1–3–4–7–8	10+5+20+6 = 41
1–3–6–7–8	10+12+7+6 = 35
1–3–6–8	10+12+7 = 29

Critical path 1–2–4–7–8 = 42 days

The new formulation of the problem is as follows:

(i) Activities 1-2, 1-3 and 1- 4 need 9, 10 and 6 days respectively as per original programme.

(ii) Activity 2-4 needs 15 + 3-9 = 9 days instead of original programme of 7 days.

(iii) Activity 3-6 needs 15 + 18 -10 = 23 days.

(iv) Activity 3-4 needs 5 days.

(v) Activities 2-5, 4-7 and 5-7 need 18, 20 and 8 days respectively.

(vi) Activity 6-7 needs 12 days as no work was scheduled to be started for this activity on 15th day.

(vii) Activities 6-8 and 7-8 need 5 and 6 days respectively.

Revised

Path	Duration
1–2–5–7–8	9+18+8+6 = 41
1–2–4–7–8	9+9+20+6 = 44
1–4–7–8	6+20+6 = 32
1–3–4–7–8	10+5+20+6 = 41
1–3–6–7–8	10+23+12+6 = 51
1–3–6–8	10+23+5 = 38

Critical path 1–3–6–7–8 = 51 days

The new diagram based on the above listed activities will be as follows:

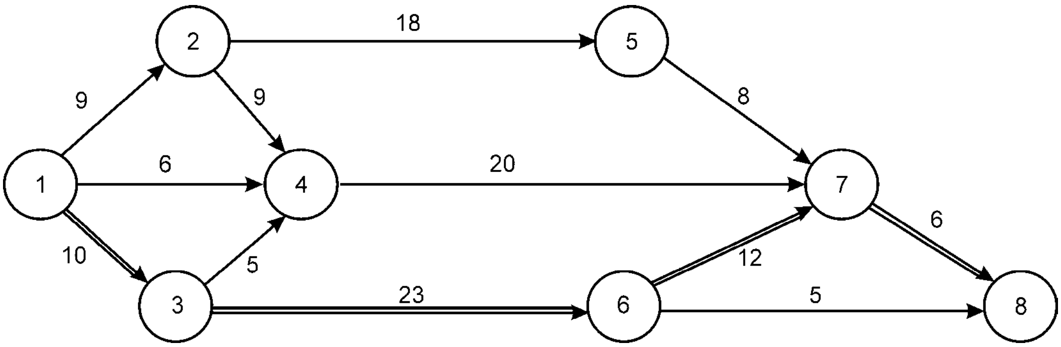


Figure 6

14.3 Project Crashing

In the discussion on PERT, we saw how the probability of completion of a project can be computed for a specified duration. There are usually compelling reasons to complete the project earlier than the originally estimated duration of the critical path computed on the basis of normal activity times, by employing extra resources. An example would be introduction of a new project. The motive in hastening the project might be to ensure that the competitors do not steal a march. In the present section we will deal with those situations which will speak of the effect of increase or decrease in the total duration for the completion of a project and are closely associated with cost considerations. In such cases when the time duration is reduced, the project cost increases, but in some exceptional cases project cost is reduced as well. The reduction in cost occurs in the case of those projects which make use of a certain type of resources, for example, a machine and whose time is more valuable than the operator's time. Before we take up an example of project cost control, it is better to understand well the following preliminaries and their definitions.

Activity Cost : It is defined as the cost of performing and completing a particular activity or task.

Crash Cost Cc	:	This is the direct cost that is anticipated in completing an activity within the crash time.
Crash time, Ct	:	This is the minimum time required to complete an activity.
Normal Cost, Nc	:	This is the lowest possible direct cost required to complete an activity.
Normal time, Nt	:	This is the minimum time required to complete an activity at normal cost.

Activity cost slope: The cost slope indicates the additional cost incurred per unit of time saved in reducing the duration of an activity. It can be understood more clearly by considering the figure 7.

Let OA represent the normal time duration for completing a job and OC the normal cost involved to complete the job. Assume that the management wishes to reduce the time of completing the job to OB from normal time OA. Therefore under such a situation the cost of the project increases and it goes up to say OD (Crash Cost). This only amounts to saving that by reducing the time period by BA the cost has increased by the amount CD. The rate of increase in the cost of activity per unit with a decrease in time is known as cost slope and is described as below.

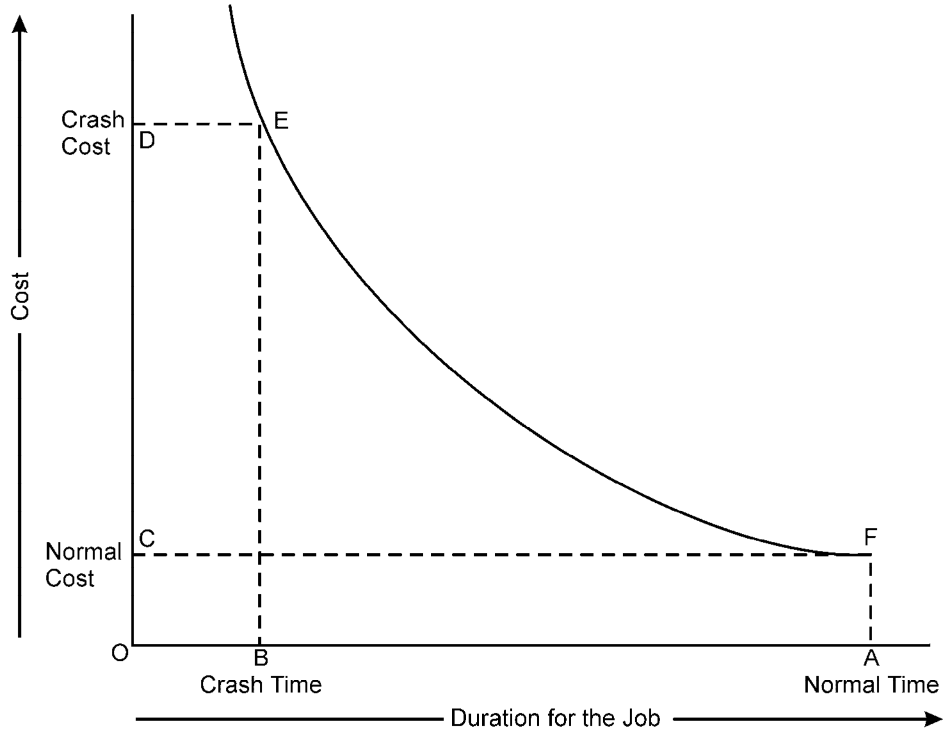


Figure 7

$$\text{Activity Cost Slope} = \frac{CD}{AB} = \frac{OD-OC}{OA-OB}$$

$$= \frac{\text{Crash Cost} - \text{Normal Cost}}{\text{Normal Time} - \text{Crash Time}}$$

Optimum duration: The total project cost is the sum of the direct and the indirect costs. In case the direct cost varies with the project duration time, the total project cost would have the shape indicated in the following figure:

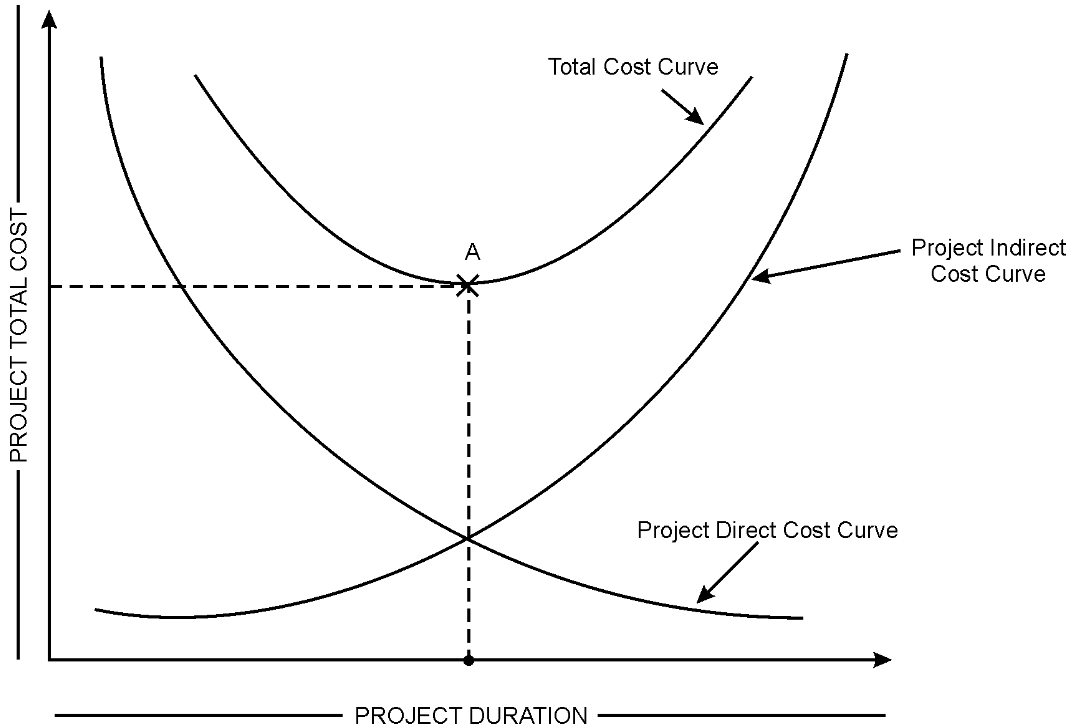


Figure 8

At the point A, the cost will be minimum. The time corresponding, to point A is called the optimum duration and the cost as optimum cost for the project.

The example below which the student should go through carefully is intended to explain cost implications of hastening a project.

Illustration 7

The following data pertains to the network drawn in figure 9 (A) given on the next page. It is desired to compress the project to the least possible duration day by day and estimate the extra cost.

i–j	T_n	T_e	*Cost slope*
1–2	*3*	*2*	*700*
1–3	*7*	*4*	*200*
2–3	*5*	*3*	*100*
2–4	*8*	*6*	*200*
3–4	*4*	*2*	*400*

Solution

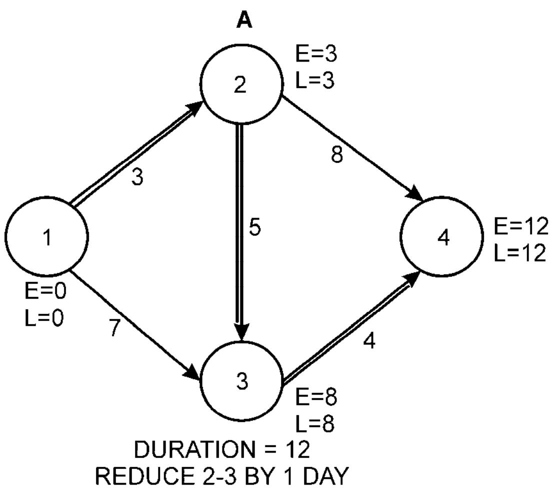
The critical path is 1-2-3-4 in figure 9 (A). It can be, seen that the critical path 1-2-3-4 is longer than either of the paths; 1-3-4 and 1-2-4 by one day. Therefore, the project can be compressed by one day along the critical path; 2-3 having the least cost slope is therefore crashed by a day.

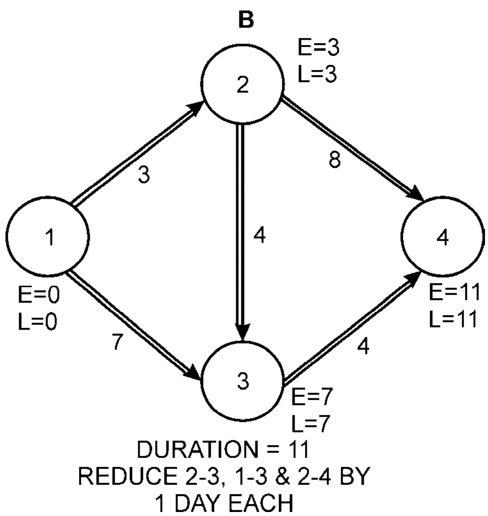
The revised network is depicted in figure 9 (B) where all activities have become critical. The following choices of compression exist now. Each set of activities is so chosen that it *reduces all the paths by a day*.

Crash each activity in one of the following sets by a day.

1–2	2–4	1–3
1–3	3–4	2–3
		2–4
Cost = 900	Cost = 600	Cost = 500

The last set of crashing 1-3, 2-3 and 2-4 is the least expensive and these activities are crashed accordingly.





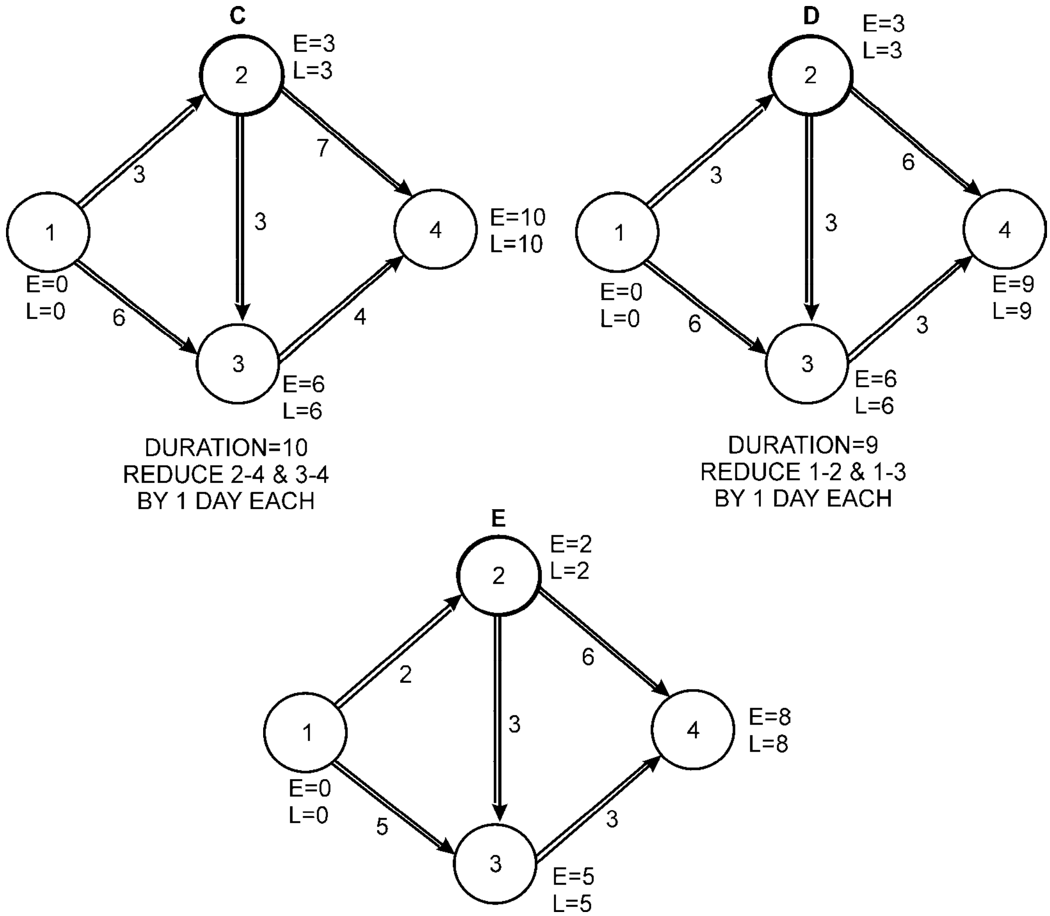


Figure 9 [(A) to (E)]

Extra cost	= 200
	100
	200
	₹ 500

This is shown in figure 9 (C). Activity 2-3 having been crashed to the limit is dropped out from further consideration. In 9 (C). the following choices of crashing each activity a day exist:

	1–3	2–4
	1–2	3–4
Extra cost	₹ 900	₹ 600

Extra cost the last is selected (Extra cost = ₹ 600)

The revised network is shown in figure 9 (D) where 2-4 joins 2-3 in that it is also crashed to limit. The only possibility of compressing the network in figure 9 (D) is to crash 1-2 and 1-3 by a day each. This is done and the final network is shown in figure 9 (E).

Extra cost = ₹ 900

Although 1-3 and 3-4 have not reached their crashing limits in figure 9 (E) there is no use also to crash these since this would not compress the project which can be compressed to 8 days only.

Total extra cost = ₹ 100 + ₹ 500 + ₹ 600 + ₹ 900 =₹ 2,100.

However, If just the least duration plan was required one could go about the problem in a much simpler way as follows. Draw the network with t_e's. This is done in the figure 9 (F).

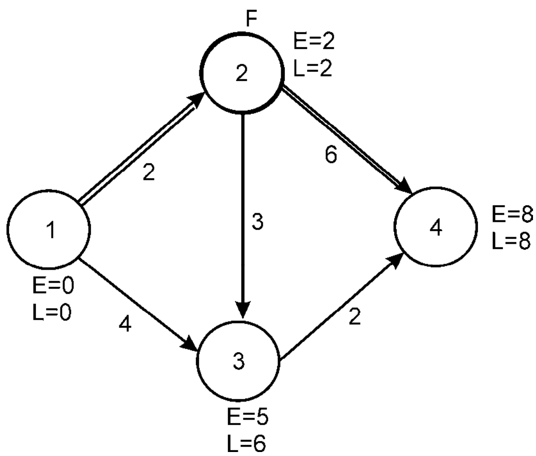


Figure 9(F)

The network is analysed with 1-2-4 as the critical path of 8 day's duration. The other paths have to be contracted to 8 day's duration. This can be done in one of the following ways:

(i) Increase 1-3 and 3-4 by a day each with cost reduction of ₹ 200 + ₹ 400 = ₹ 600.

(ii) Increase 2-3 and 1-3 by a day and 2 days respectively with cost reduction of ₹ 100 + ₹ 400 = ₹ 500.

Obviously the 1st course is to be preferred and the network. if now revised. would be identical with the one of figure 9 (E).

However, management would be interested in the least total cost duration rather than the least possible duration.

Now, suppose there is an indirect cost of ₹ 800 per day. What would then be the least cost project duration for the example on hand ? The various cost data are tabulated below:

Duration	Direct (Crashing) cost	Indirect cost	Total cost
12	0	9,600	9,600
11	100	8,800	8,900
10	600	8,000	8,600
9	1,200	7,200	8,400
8	2,100	6,400	8,500

Thus 9 days is the least total cost duration and rationally the management should go in for this unless high opportunity losses compel them to select a lower duration project plan.

Illustration 8

Consider the following network and the table for a particular project which consists of 7 activities.

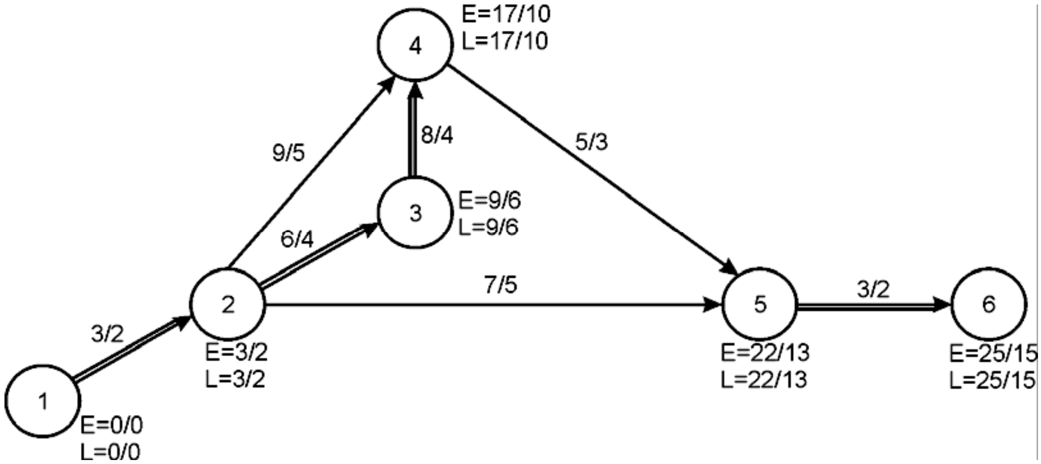


Figure 10

(The number indicated along the activity arrows are the normal duration.)

Activity	*Normal*		*Crash*		*Cost slopes*		
	Times (days)	*Cost ₹*	*Time (days)*	*Cost ₹*	*ΔT*	*ΔC*	*ΔC/ΔT*
1–2	*3*	*360*	*2*	*400*	*1*	*40*	*40*
2–3	*6*	*1,440*	*4*	*1,620*	*2*	*180*	*90*
2–4	*9*	*2,160*	*5*	*2,380*	*4*	*220*	*55*
2–5	*7*	*1,120*	*5*	*1,600*	*2*	*480*	*240*
3–4	*8*	*400*	*4*	*800*	*4*	*400*	*100*
4–5	*5*	*1,600*	*3*	*1,770*	*2*	*170*	*85*
5–6	*3*	*480*	*2*	*760*	*1*	*280*	*280*
		7,560		*9,330*			

The indirect cost is ₹160/- per day. Determine optimum duration in the above case.

Solution

The normal duration of the project is obtained from the critical path and not by merely summing up the normal duration of all the activities. The overall normal duration in the above project is 25 days and the total direct cost is ₹ 7,560/-. If due to some reason it is felt that the project is to be hastened, the question then to be answered is: “what is the minimum time required for the completion of the project?”

According to the crash time given in the table; the critical path based on these crash time estimates still appears as 1-2-3-4-5-6 and the crash duration is 15 days. Hence, the project may take 15 to 25 days depending upon the money the management is prepared to spend. If the management decides to complete the project in 15 days, then the direct cost will be ₹ 9330/-. The figure has been arrived by speeding up or crashing all the activities as mentioned in the, given table. But there are several non-critical activities in the project, which need not be crashed to reduce the project duration. Speeding up of such non critical activities involves extra amount. As any extra amount spent on these activities is not going to reduce the project duration, therefore it is better not to reduce the duration of these non-critical activities. Duration of these non-critical activities should be reduced only when they become critical activities, during the process of reducing the project duration time.

The two objectives behind the reduction of project time are:

(1) To complete the project before a certain target date.

(2) To reduce the overall cost of the project. This objective can only be achieved when the indirect cost per day is greater than some of the cost slopes as given in the table. In the exercise under consideration, the five activities 1-2, 2-3, 2-4, 3-4 and 4-5 have cost slopes lower than the indirect cost. In such a situation, the management would be very much interested in cutting down the project time, thereby decreasing the total indirect cost. In order to solve this example we proceed as follows:

First step is to identify those activities along the critical path whose cost slopes are less than the indirect cost. In our network such activities are 1-2,2-3,3-4 and 4-5. We take these activities in the order of increasing cost slopes. Activity 1-2 has a slope of ₹ 40/per day. This activity can be cut down by 1 day at a cost of ₹ 40/-. The next activity in the order of cheapness is 4-5. This can be cut down by 2 days at a cost of ₹ 170. Next we may take the activity 2-3 whose cost slope is ₹ 90/-. This activity can be contracted by 2 days at a cost of ₹ 180. So far three activities viz. 1-2,4-5 and 2-3 have been contracted by a total of 5 days at an overall cost of ₹ 390/-.

From the given table, we notice that the next activity in the order of priority is 3-4 with a cost slope ₹ 100/- per day. This activity can be cut down by 4 days. But a look on the network after the performance of above reduction in time reveals that activity 2-4 has float of only 3 days.

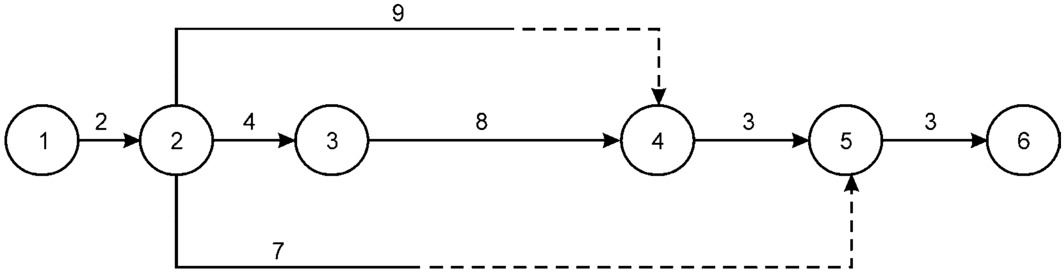


Figure 11

Hence we cannot cut down the duration for the activity 3-4 by four days without affecting the activity 2-4. Therefore one should cut down the duration on activity 3-4 by 3 days at a cost of

₹ 300/- and now by this process a new sub-critical path viz. 1-2-4-5-6 may come into existence.

According to the given Table, activity 3-4 can be further contracted by 1 day. But this cannot be achieved without contracting activity 2-4. Therefore, the cost slope must include both of these activities i.e. activities 2-4 and 3-4. The combined cost slope of these : two activities is ₹ 55 + ₹ 100 = ₹ 155. The project duration has now been reduced to 16 days, as below:

Activity	Duation reduced by (Days)	Cost slope (₹)	Cost of contraction (₹)
1–2	1	40/-	40/-
4–5	2	85/-	170/-
2–3	2	90/-	180/-
3–4	3	100/-	300/-
3–4,2–4	1	155/-	155/-
Total	9		845/-

During the above process a total reduction of 9 days has been achieved at an extra cost of ₹ 845/-. Also the indirect cost @ ₹ 160/- per day for 9 days resulting in ₹ 1,440/- has been saved.

In other words, the project duration has been reduced from 25 days to 16 days. As a result of this duration of project its direct cost has now become ₹ 8,405/- and ₹ 2,560/- as its indirect cost. The overall project cost with the new time schedule i.e. 16 days is ₹ 8,405 + ₹ 2,560 = ₹ 10,965/- as against the overall cost of ₹ 11,560, it thus results in a net saving of ₹ 595/-.

Illustration 9

The Arcot Machinery Company has been offered a contract to build and deliver nine extruding presses to the Home Botting Company. The contract price negotiated is contingent upon meeting a specified delivery time, with a bonus offered for early delivery. The marketing department has established the following cost and time information:

Activity	*Normal time (weeks)*			*Normal Cost*	*Crash Time (Weeks)*	*Crash Cost*
	a	*b*	*m*	*(₹)*		*(₹)*
1–2	*1*	*5*	*3*	*5,000*	*1*	*9,000*
2–3	*1*	*7*	*4*	*8,000*	*3*	*14,000*
2–4	*1*	*5*	*3*	*4,000*	*2*	*6,000*
2–5	*5*	*11*	*8*	*5,000*	*7*	*6,000*
3–6	*2*	*6*	*4*	*3,000*	*2*	*5,000*
4–6	*5*	*7*	*6*	*2,000*	*4*	*3,600*
5–7	*4*	*6*	*5*	*10,000*	*4*	*14,000*
6–7	*1*	*5*	*3*	*7,000*	*1*	*10,600*

Normal delivery time is 16 weeks for a contract price of ₹ 62,000.

On the basis of the calculated profitability for each delivery time specified in the following table, what delivery schedule do you recommend that the company may implement?

Contract Delivery Time (weeks)	*Contract Amount (₹)*
15	*62,500*
14	*65,000*
13	*70,000*
12	*72,500*

(Here a = t_o : optimistic time, b=t_p; pessimistic time, m=t_m; most likely time.)

Solution

Let us first calculate the expect duration of each activity.

Activity	Normal Time (weeks)			Expected duration (weeks)	Normal Cost (₹)	Crash Time (weeks)	Crash Cost (₹)	Cost Slope (₹)
	a	b	m					
1-2	1	5	3	3	5,000	1	9,000	2,000
2-3	1	7	4	4	8,000	3	14,000	6,000
2-4	1	5	3	3	4,000	2	6,000	2,000
2-5	5	11	8	8	5,000	7	6,000	1,000
3-6	2	6	4	4	3,000	2	5,000	1,000
4-6	5	7	6	6	2,000	4	3,600	800
5-7	4	6	5	5	10,000	4	14,000	4,000
6-7	1	5	3	3	7,000	1	10,600	1,800
					44,000			

The network for the given problem is drawn below:

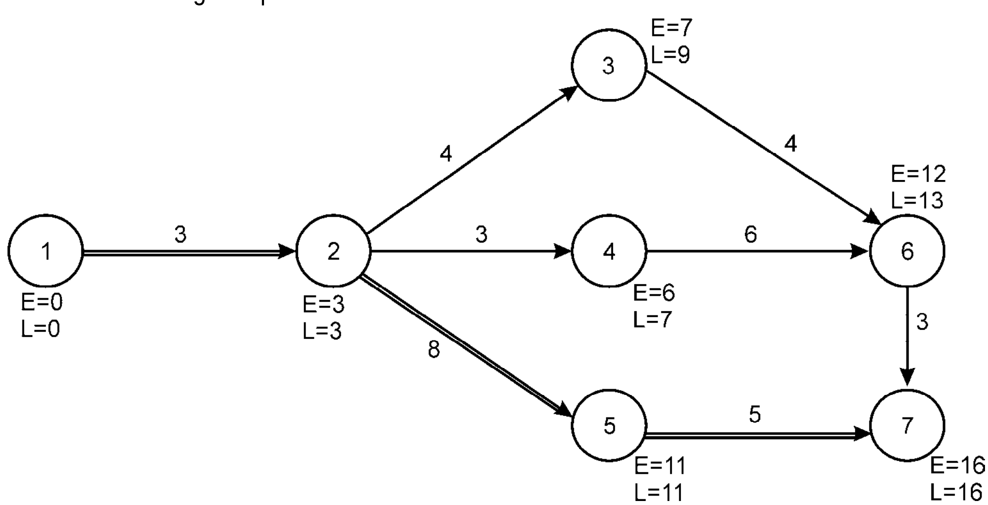


Figure 12

The critical path is 1-2-5-7 with total duration 16 weeks. Cost of all activities is ₹44,000. Contract price is ₹ 62,000 for normal delivery time of 16 weeks. Hence the profit is of ₹ 18,000.

For calculating the most profitable delivery schedule, let us start crashing the activities on the critical path. Cost slopes for various activities are given in the above table.

Step 1: The critical activity 2-5 has the least cost slope of ₹ 1,000, so we crash this activity by 1 day. Crashing cost is ₹ 1,000. The contract amount for delivery time of 15 weeks is ₹ 62,500

Profit = ₹ 62,500 - (₹ 44,000 + ₹ 1,000)

= ₹ 62,500 -₹ 45,000 = ₹ 17,500

Step 2: Now there are two critical paths viz. 1-2-4-6-7 and 1-2-5-7 with duration of 15 weeks. So we crash activity 1-2 by 1 day at crash cost of ₹ 2,000. Project duration is 14 weeks.

Profit = ₹ 65,000 - ₹ (45,000 + 2,000) = ₹ 18,000.

Step 3: We again crash activity 1-2 by 1 day. So the project duration is 13 weeks

Profit = ₹ 70,000 - (₹ 47,000 + ₹ 2,000)

= ₹ 70,000 - ₹ 49,000 = ₹ 21,000

Step 4: Now we crash activity 4-6 for path 1-2-4-6-7 at crash cost of ₹ 800 and activity 5-7 for path 1-2-5-7 at crash cost of ₹ 4000. Project duration is now 12 weeks.

Profit = ₹ 72,500 - (₹ 49,000 + ₹ 4000 + ₹ 800)

= ₹ 72,500 - ₹ 53,800 = ₹ 18,700

No further crashing is possible.

From step 3, it can be seen that the profit is maximum when the project duration is 13 weeks. Hence, the company should implement the delivery schedule of 13 weeks at a contract amount of ₹ 70,000 to gain maximum profit of ₹ 21,000

14.4 Resource Smoothing

It is a network technique used for smoothening peak resource requirement during different periods of the project network. Under this technique the total project duration is maintained at the minimum level. For example, if the duration of a project is 15 days, then the project duration is maintained, but the resources required for completing different activities of a project are smoothened by utilising floats available on non critical activities. These non critical activities having floats are rescheduled or shifted so that a uniform demand on resources is achieved. In other words, the constraint in the case of resource smoothing operation would be on the project duration time. Resource smoothing is a useful technique or business managers to estimate the total resource requirements for various project activities.

In resources smoothing, the time-scaled diagram of various activities and their floats (if any), along with resource requirements are used. The periods of maximum demand for resources are identified and non critical activities during these periods are staggered by rescheduling them according to their floats for balancing the resource requirements.

Before explaining the process, let us first illustrate the concept of ' Time Scaled Diagram' which we will be using in the subsequent problems:

Time Scaled Diagrams: In the network diagrams which we have considered, it has been stressed that the length of the individual arrows has no relation to the duration of the activity which each arrow represented. It is of course possible to draw the arrows to a time scale, and this can be a very useful method of presentation for small networks.

The figure 11 shows the network diagram drawn to a horizontal time scale. The critical path has been arranged as a straight line with non-critical events above or below it. Solid lines represent activities, dotted horizontal lines represent float, and dotted vertical lines represent dummies. The presentation has obvious advantages. The events are entered in the time scale version at the earliest start time. (The latest start times can also be used as an alternative).

Illustration 10

Consider a project consisting of thirteen activities having duration and resource requirements shown below:

Activity	*Duration (days)*	*Labourers required*
A	*2*	*2L*
B	*4*	*2C,2L*
C	*4*	*4C*
D	*3*	*2L*
E	*4*	*6C*
F	*7*	*2L*
G	*2*	*4C*
H	*5*	*4C,2L*
I	*2*	*2C*
J	*5*	*2C*
K	*2*	*2L*
L	*3*	*4L*
M	*2*	*4L*

Here L stands for labourers and C stands for carpenters, we shall analyse this project from the point of view of resources to bring out the necessary steps involved in the analysis and smoothing of resources.

From the values given in table above E and L are calculated and given in the network (Fig. 13)

The critical path for this network is 1–2–4–6–8–9–10–11, in figure 14 is shown the time-scaled version of the same network. The critical path is shown along the horizontal line. The last two rows (also referred to as resource accumulation table) in figure 14 give the number of labourers and carpenters required each day. It can be seen that the demand on the resources is not even. On the 7th and 8th days the demand for carpenters is as high as 14, whereas on the 11th, 12th and 13th days it is two only. If the carpenters and labourers are to be hired for the entire project duration of 22 days, then during most of the days they will be idle and the company will have to hire at least 14 carpenters and 4 labourers.

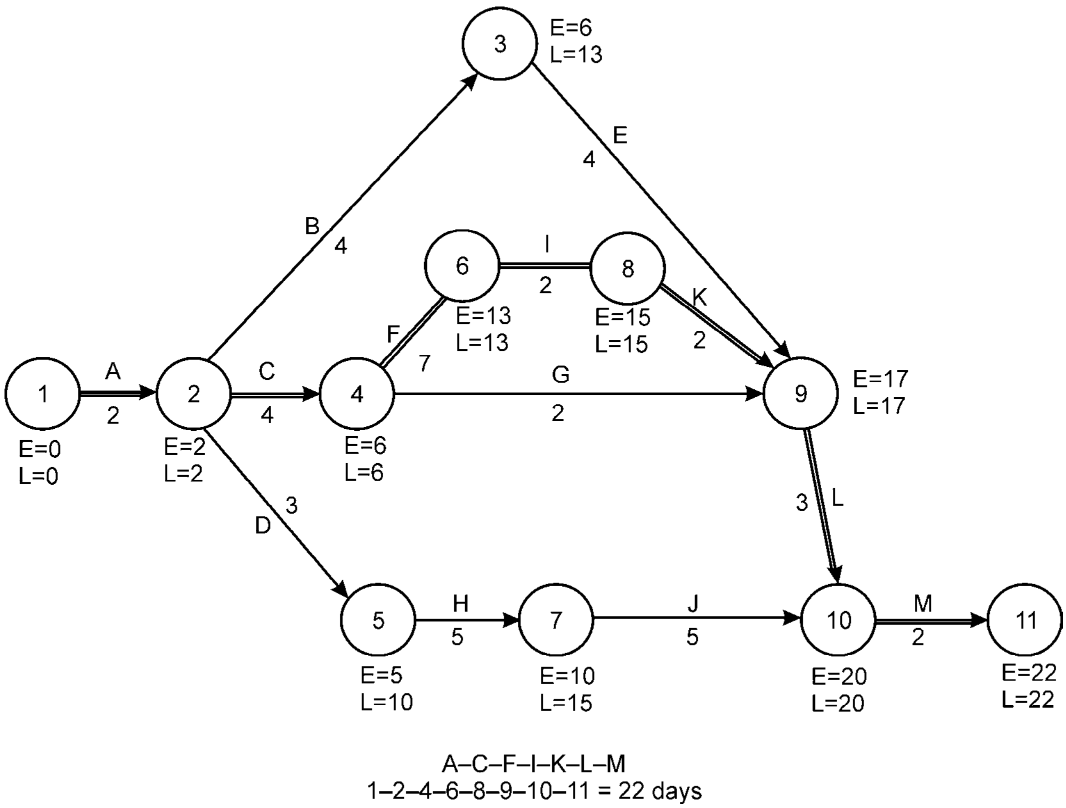


Figure 13

Activity	Duration	EST	EFT =EST +D	LST= LFT–D	LFT	Slack of Tail Event	Slack of Head Event	Total Float =LST –EST	Free Float = Total Float - Slack of Head Event	Independent Float=Free Float–Slack of Tail Event
A(1–2)	2	0	2	0	2	0	0	0	0	0
B(2–3)	4	2	6	9	13	0	7	7	0	0
C(2–4)	4	2	6	2	6	0	0	0	0	0
D(2–5)	3	2	5	7	10	0	5	5	0	0
E(3–9)	4	6	10	13	17	7	0	7	7	0
F(4–6)	7	6	13	6	13	0	0	0	0	0
G(4–9)	2	6	8	15	17	0	0	9	9	9
H(5–7)	5	5	10	10	15	5	5	5	0	0*
I(6–8)	2	13	15	13	15	0	0	0	0	0
J(7–10)	5	10	15	15	20	5	0	5	5	0
K(8–9)	2	15	17	15	17	0	0	0	0	0
L(9–10)	3	17	20	17	20	0	0	0	0	0
M(10–11)	2	20	22	20	22	0	0	0	0	0

* If negative value is obtained, the independent float is taken to be zero.

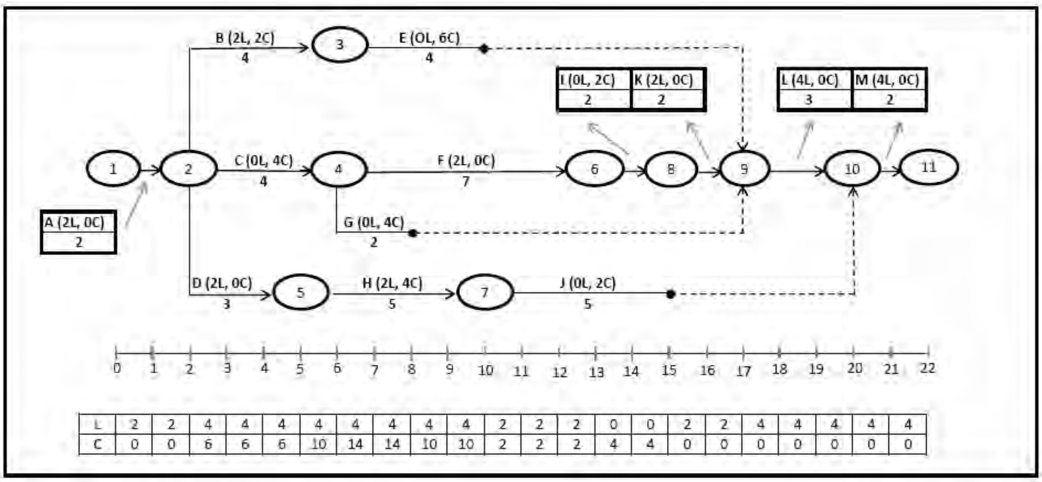
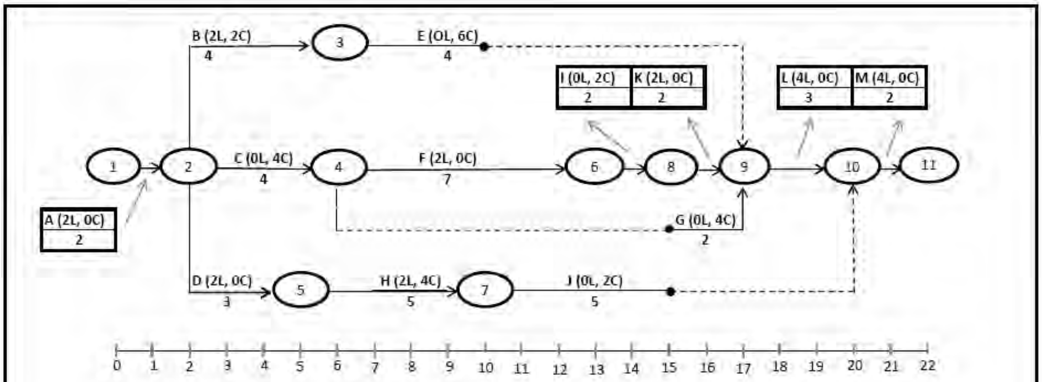


Figure 14

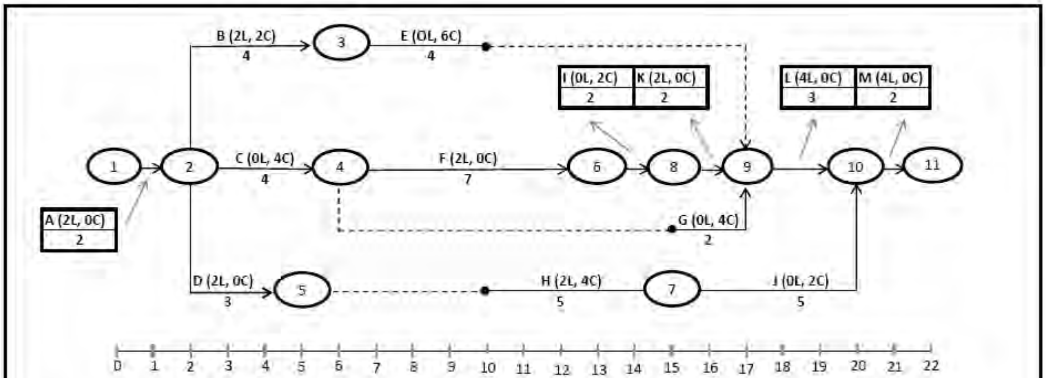
We will attempt to re-schedule our activities in-such a way as to utilize the resources in a fairly uniform manner. As mentioned above the maximum demand on the resources occurs on the 6th, 7th, 8th, 9th and 10th days. The activities on these days will have to be shifted depending upon their floats such that the demand comes down. As can be seen from the above time-scaled version, activity 4-9 has maximum float, therefore we will try to shift activity 4-9 so that is starts on the 16th day instead of the 7th day. This reduces the demand on the carpenters from 14 to 10 on the 7th and 8th days so that the maximum demand for the carpenters on any day is now 10 and not 14. The modified resource accumulation table and the time-scaled version of the project is shown in figure 15.



L	2	2	4	4	4	4	4	4	4	4	2	2	2	0	0	2	2	4	4	4	4	4
C	0	0	6	6	6	10	10	10	10	10	2	2	2	4	4	4	4	0	0	0	0	0

Figure 15

It is evident from the above figure that the maximum demand now is for 10 carpenters on the 6th, 7th, 8th, 9th and 10th days. We will now try to explore that possibility of further smoothing the resources which is possible because activities 2–5, 5–7 and 7–10 have a total float of 5 days. The resultant time-scaled network and the resource accumulation table are given below in fig. 16.



L	2	2	4	4	4	2	2	2	2	2	4	4	4	2	2	2	2	4	4	4	4	4
C	0	0	6	6	6	6	6	6	6	6	4	4	4	6	6	6	6	2	2	2	0	0

Figure 16

As can be seen from the above figure the requirement for labourers is 4 and the requirement for carpenters reduces to 6 as against 14 carpenters originally estimated. Hence, by judiciously utilizing the float, we can smooth the demand on the resources.

14.5 Resource Levelling

It is also a network technique which is used for reducing the requirement of a particular resource due to its paucity. The process of resource levelling utilizes the large floats available on non-critical activities of the project and thus cuts down the demand on the resource. In resource levelling, the maximum demand of a resource should not exceed the available limit at any point of time. In order to achieve this, non critical activities are rescheduled by utilising their floats. *Sometimes, the use of resource levelling may lead to enlonging the completion time of the project*. In other words, in resource levelling, constraint is on the limit of the resource availability.

14.6 Miscellaneous Illustrations

Illustration 11

A project consists of eight activities with the following relevant information:

Activity	*Immediate Predecessor*	*Estimated Duration (Days)*		
		Optimistic	*Most Likely*	*Pessimistic*
A	*–*	*1*	*1*	*7*
B	*–*	*1*	*4*	*7*
C	*–*	*2*	*2*	*8*
D	*A*	*1*	*1*	*1*
E	*B*	*2*	*5*	*14*
F	*C*	*2*	*5*	*8*
G	*D,E*	*3*	*6*	*15*
H	*F,G*	*1*	*2*	*3*

(i) Draw the PERT network and find out the expected project completion time.

(ii) What duration will have 95% confidence for project completion?

(iii) If the average duration for activity F increases to 14 days, what will be its effect on the expected project completion time which will have 95% confidence?

Solution

(i) The required network is drawn below:

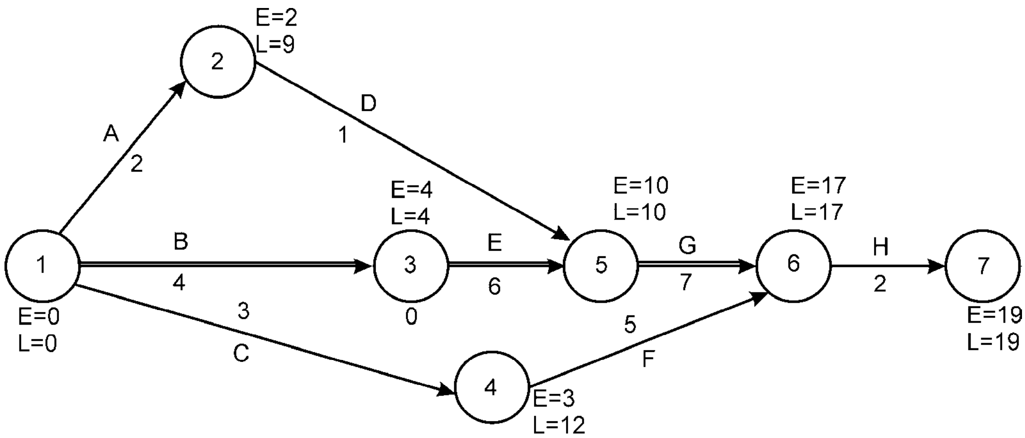


Figure 17

The expected time for each activity shown in the network above is calculated in the following table:

Activity		Estimated Duration (Days)			Expected Duration $\left(\frac{a+4m+b}{6}\right)$	Variance $\left(\frac{b-a}{6}\right)^2$
		Optimistic a	Most likely m	Pessimistic b		
A	1–2	1	1	7	2	1
B	1–3	1	4	7	4	1
C	1–4	2	2	8	3	1
D	2–5	1	1	1	1	0
E	3–5	2	5	14	6	4
F	4–6	2	5	8	5	1
G	5–6	3	6	15	7	4
H	6–7	1	2	3	2	1/9

The critical path is given by 1-3-5-6-7 or B-E-G-H and the expected project completion time is 19 days.

(ii) The variance for critical path is 1 + 4 + 4 + 1/9 = 82/9

Standard deviation of critical path = $\sigma_1 = \sqrt{\frac{82}{9}}$ = 3.02 (approx.)

To calculate the project duration which will have 95% chances of its completion, we utilise the value of Z corresponding to 95% confidence which is 1.645.

Thus, $\frac{X-19}{3.02} = 1.645$,

or X = 1.645 × 3.02 + 19 = 23.97 days = 24 days

Hence, 24 days of project completion time will have 95% probability of its completion.

(iii) If the average duration for activity F increases to 14 days, then the path 1-4-6-7 i.e., C-F-H will also become critical path with expected project completion time of 19 days. Now, activities C and F are also critical activities.

Variance of Critical path C – F – H = 1 + 1 + 1/9 = 19/9

Variance of Critical path B – E – G – H = 1 + 4 + 4 + 1/9 = 82/9

$$\sigma = \sqrt{\frac{82^*}{9}} = 3.02 \text{ (appx)}$$

* If there are two equal longest path, Higher of the two would be picked up.

We now wish to calculate the expected project completion time that will have 95% confidence level,

P (Z < 1.645) = 0.95

$$\frac{X-\mu}{\sigma} = \frac{X-19}{3.02} = 1.645$$

or X = 23.97 days.

Hence the project duration of 23.97 days will have 95% confidence of completion.

Illustration 12

The following information is given below:

Activity	*(1–2)*	*(2–3)*	*(2–4)*	*(3–5)*	*(4–6)*	*(5–6)*	*(5–7)*	*(6–7)*
Pessimistic time (in weeks)	*3*	*9*	*6*	*8*	*8*	*0*	*5*	*8*
Most likely time (in weeks)	*3*	*6*	*4*	*6*	*6*	*0*	*4*	*5*
Optmistic time (in weeks)	*3*	*3*	*2*	*4*	*4*	*0*	*3*	*2*

Draw the network diagram for the above.

Calculate:

(i) Variance to each activity.

(ii) Critical path and expected project length.

(iii) The probability that the project will be completed in 23 weeks is drawn below:

If there are two equal longest path higher of the two variance would be picked up.

Solution

The required network is drawn below:

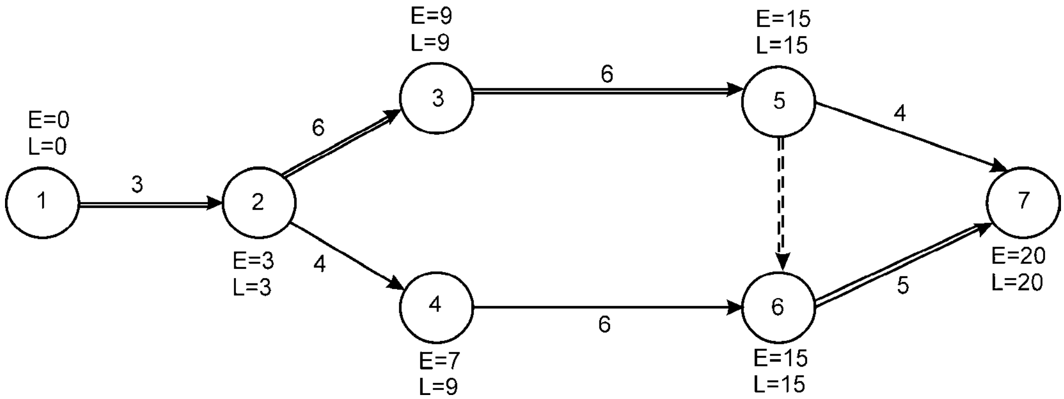


Figure 18

The expected time marked in the above network diagram for various activities is calculated in the table below:

Activities	Times (in weeks)			Expected time (weeks) $t_e = \frac{t_o + 4t_m + t_p}{6}$	Variance $\left(\frac{t_p - t_o}{6}\right)^2$
	Optimistic (t_o)	Most like (t_m)	Pessimistic (t_p)		
1–2	3	3	3	3	0
2–3	3	6	9	6	1
2–4	2	4	6	4	4/9
3–5	4	6	8	6	4/9
4–6	4	6	8	6	4/9
5–6	0	0	0	0	0
5–7	3	4	5	4	1/9
6–7	2	5	8	5	1

(i) Variance of each activity has been calculated in the last column of the above table.

(ii) Critical path is given by 1–2–3–5–6–7 and the expected project length is 20 weeks.

(iii) Variance of the critical path = σ^2 = 0 + 1 + 4/9 + 0 + 1 = 22/9 = 2.444

Mean = x = 20 weeks

To calculate the probability of completing the project in 23 weeks, we will first calculate the normal Z as below:

$$Z = \frac{D-x}{\sigma} = \frac{23-20}{\sqrt{2.444}} = 1.92.$$

P (x $\leq$ 23) = P (z $\leq$ 1.92) = 0.9726 (from the Normal table)

Thus, the probability that the project will be completed in 23 weeks is 97.26%.

Illustration 13

A small maintenance project consists of the following twelve jobs whose precedence relations are identified with their node number:

Job (i–j)	*(1,2)*	*(1,3)*	*(1,4)*	*(2,3)*	*(2,5)*	*(2,6)*
Duration (in days)	*10*	*4*	*6*	*5*	*12*	*9*
Job (i–j)	*(3,7)*	*(4,5)*	*(5,6)*	*(6,7)*	*(6,8)*	*(7,8)*
Duration (in days)	*12*	*15*	*6*	*5*	*4*	*7*

(i) *Draw an arrow diagram representing the project.*

(ii) *Calculate earliest start, earliest finish, start and latest finish time for all jobs.*

(iii) *Find the critical path and project duration.*

(iv) *Tabulate total float, free float and independent float.*

Solution

(i) The required arrow diagram is drawn in below:

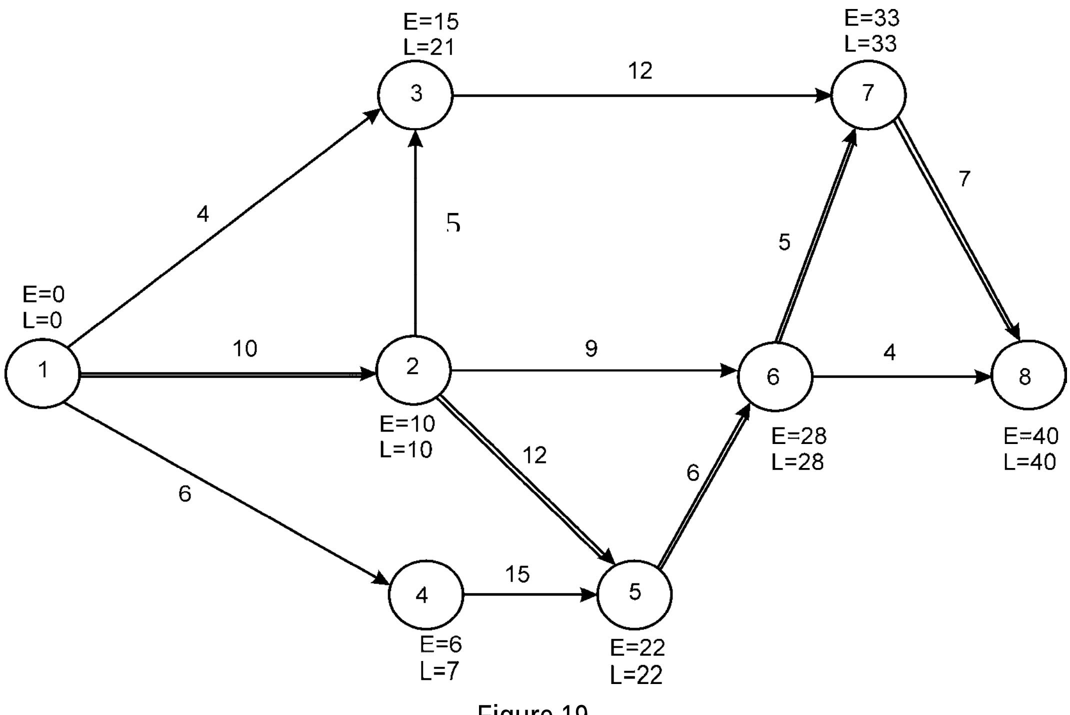


Figure 19

(ii) The earliest start, earliest finish, latest start and latest finish time for all the jobs are calculated in the table given below:

Job (i–j)	*Duration in Days*	*Earliest time*		*Latest time*		*Slack of*		*Total Float =(LST-EST)*	*Free Float*	*Indep. Float*
		Start EST	*Finish EFT*	*Start LST*	*Finish LFT*	*Tail Event*	*Head Event*			
1–2	10	0	10	0	10	0	0	0	0	0
1–3	4	0	4	17	21	0	6	17	11	11
1–4	6	0	6	1	7	0	1	1	0	0
2–3	5	10	15	16	21	0	6	6	0	0
2–5	12	10	22	10	22	0	0	0	0	0
2–6	9	10	19	19	28	0	0	9	9	9
3–7	12	15	27	21	33	6	0	6	6	0
4–5	15	6	21	7	22	1	0	1	1	0
5–6	6	22	28	22	28	0	0	0	0	0
6–7	5	28	33	28	33	0	0	0	0	0
6–8	4	28	32	36	40	0	0	8	8	8
7–8	7	33	40	33	40	0	0	0	0	0

(iii) The critical path is 1-2-5-6-7-8 and the project duration is 40 days.

(iv) Total float, free float and independent float for various activities are calculated in the above table.

Illustration 14

A small project consists of seven activities for which the relevant data are given below:

Activity	*Preceding Activities*	*Activities Duration (Days)*
A	–	*4*
B	–	*7*
C	–	*6*
D	*A,B*	*5*
E	*A,B*	*7*
F	*C,D,E*	*6*
G	*C,D,E*	*5*

(i) Draw the network and find the project completion time.

(ii) Calculate total float for each of the activities.

(iii) Draw the time scaled diagram.

Solution

(i) The required network is given in below:

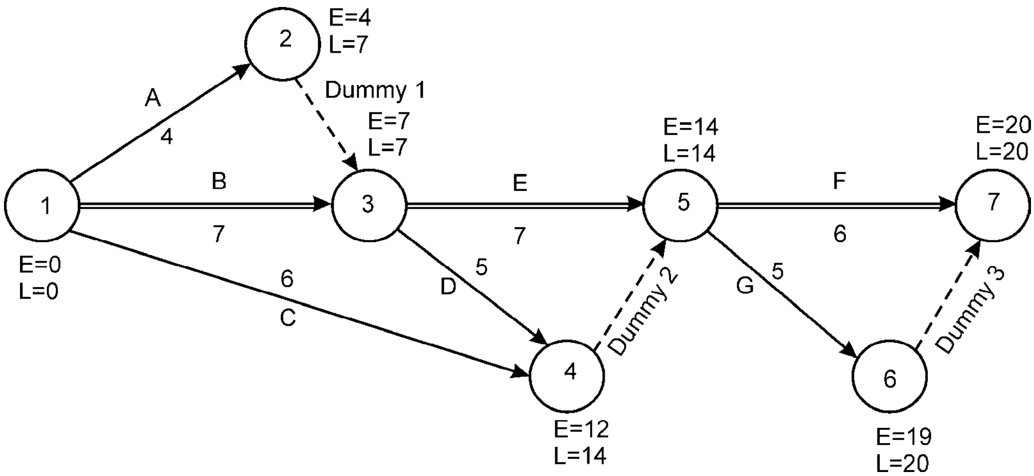


Figure 20

The critical path is 1–3–5–7 with duration of 20 days. Thus, the project completion time is 20 days.

(ii) The total float for various activities is calculated as below:

Activity		Duration	EST	LST	EFT	LFT	LST–EST
A	1–2	4	0	3	4	7	3
B	1–3	7	0	0	7	7	0
C	1–4	6	0	8	6	14	8
Dummy 1	2–3	0	4	7	4	7	3
D	3–4	5	7	9	12	14	2
E	3–5	7	7	7	14	14	0
Dummy 2	4–5	0	12	14	12	14	2
F	5–7	6	14	14	20	20	0
G	5–6	5	14	15	19	20	1
Dummy 3	6–7	0	19	20	19	20	1

(iii) The required time diagram is drawn below:

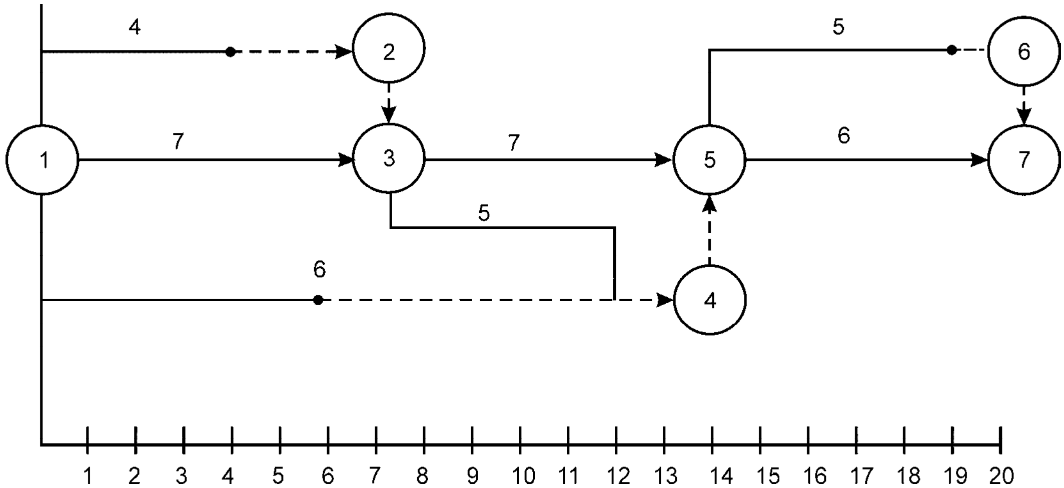


Figure 21

Illustration 15

A small project is having seven activities. The relevant data about these activties is given below:

Activity	*Dependence*	*Normal duration (Days)*	*Crash duration (Days)*	*Normal cost (₹)*	*Crash cost (₹)*
A	*–*	*7*	*5*	*500*	*900*
B	*A*	*4*	*2*	*400*	*600*
C	*A*	*5*	*5*	*500*	*500*
D	*A*	*6*	*4*	*800*	*1,000*
E	*B,C*	*7*	*4*	*700*	*1,000*
F	*C,D*	*5*	*2*	*800*	*1,400*
G	*E,F*	*6*	*4*	*800*	*1,600*

(i) *Find out the normal duration.*

(ii) *What is the percentage increase in cost to complete the project in 21 days ?*

Solution

The network is given below:

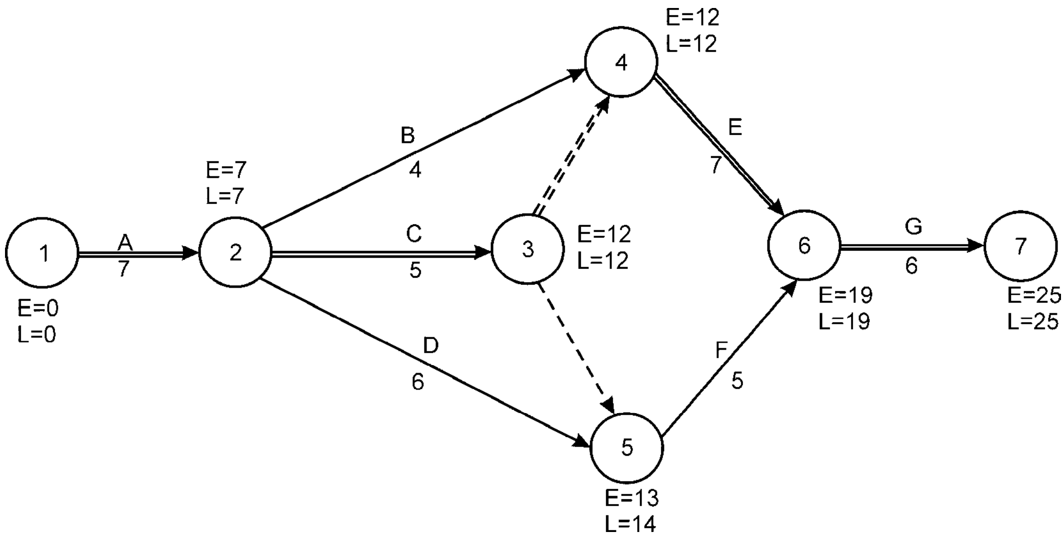


Figure 22

(i) The critical path of the project is A–C–E–G or 1–2–3–4–6–7 with normal duration of 25 days.

(ii) The cost slope for various activities is given below:

Activity Duration	Normal Duration (Days)	Crash Duration (Days)	Normal Cost (₹)	Crash Cost (₹)	Cost Slope (₹)
A(1-2)	7	5	500	900	$\frac{900-500}{7-5}=200$
B(2-4)	4	2	400	600	$\frac{600-400}{4-2}=100$
C(2-3)	5	5	500	500	N.A
D(2-5)	6	4	800	1,000	$\frac{1,000-800}{6-4}=100$
E(4-6)	7	4	700	1,000	$\frac{1,000-700}{7-4}=100$
F(5-6)	5	2	800	1,400	$\frac{1,400-800}{5-2}=200$
G(6-7)	6	4	800	1,600	$\frac{1,600-800}{6-4}=400$
Total			4,500		

Various paths of the network are given below:

1-2-3-4-6-7 with duration = 25 days

1-2-4-6-7 with duration = 24 days

1-2-3-5-6-7 with duration = 23 days

1-2-5-6-7 with duration = 24 days

Step-1:

In order to determine the cost of completing the project in 21 days, let us crash that activity on the critical path which has minimum cost slope. It can be seen that the minimum cost slope of ₹ 100 corresponds to activity E(4-6) and it lies on the critical path. Hence, we crash activity E (4-6) by 1 day at an additional cost of ₹ 100.

Various paths now are

1-2-3-4-6-7 with duration = 24 days

1-2-4-6-7 with duration = 23 days

1-2-3-5-6- 7 with duration = 23 days

1-2-5-6-7 with duration = 24 days

Step-2:

An examination of the above four paths clearly points out that there are two critical paths namely 1-2-3-4-6-7 and 1-2-5-6-7, each with duration = 24 days. To reduce the project duration by three days more, there are following possible combination of activities.

1. Crash activities 4 - 6 on the path 1-2-3-4-6-7 and 5 - 6 on the path 1-2-5-6-7 by one day each at an addition cost of ₹ 100 + ₹ 200 = ₹ 300
2. Crash activities 4-6 on path 1-2-3-4-6-7 and 2-5 on path 1-2-5-6-7 by one day each at an additional cost of ₹ 100 + ₹ 100 =₹ 200
3. Crash activity 1-2 by one day at an additional cost of ₹ 200.

It can be observed that the additional cost of reducing the project duration by one day in combination 2 as well as combination 3 is ₹ 200. Hence any of these two can be selected for crashing. However, since crashing activity 1-2 by 1 day reduces the duration of all the paths by 1 day, we will crash it by 1 day. The project duration becomes = 23 days at an additional cost = ₹ 200.

Various paths now are.

1-2-3-4-6-7 with duration = 23 days 1-2-3-5-6-7 with duration = 22 days

1-2-4-6-7 with duration = 22 days 1-2-5-6-7 with duration 23 days

Step-3:

Crash activity 1-2 by 1 day further, it would reduce the project duration to 22 days at an additional cost = ₹ 200.

Various paths now are.

1-2-3-4-6-7 with duration = 22 days 1-2-3-5-6-7 with duration = 21 days

1-2-4-6-7 with duration = 21 days 1-2-5-6-7 with duration 22 days

Step-4:

Activity 1-2 cannot be crashed further. So, we now select the combination 2 stated above for crashing. Crash activities 4-6 and 2-5 by one day each at an additional cost of ₹ 100 + ₹ 100 = ₹ 200.

Various paths now are.

1-2-3-4-6-7 with duration = 21 days 1-2-3-5-6-7 with duration = 21 days

1-2-4-6-7 with duration = 20 days 1-2-5-6-7 with duration 21 days

The project duration now becomes equal to 21 days.

Hence, in order to complete the project in 21 days, an additional cost of ₹ 100 + ₹ 200+ ₹ 200 + ₹ 200 = ₹ 700 will be incurred.

The normal cost of completing the project in 25 days = ₹ 4,500.

Hence, the percentage increase in cost to complete the project in

21 days = $\frac{₹\ 700}{₹\ 4500} \times 100 = 15.55\%$

Illustration 16

The following is a table showing details of a project.

Activity	*Immediate Predecessor*	*Normal Time in weeks*	*Normal Cost (₹ 000)*	*Crash Time in weeks*	*Crashing Cost (₹000)*
A	—	*20*	*40*	*14*	*60*
B	—	*16*	*30*	*12*	*40*
C	*B*	*10*	*16*	*8*	*28*
D	*B*	*12*	*22*	*8*	*30*
E	*B*	*16*	*18*	*10*	*30*
F	*E*	*10*	*10*	*8*	*16*
G	*A, D, C*	*24*	*6*	*16*	*8*

Indirect cost is ₹800 per day. Find the optimum duration and the associated minimum project cost.

Solution

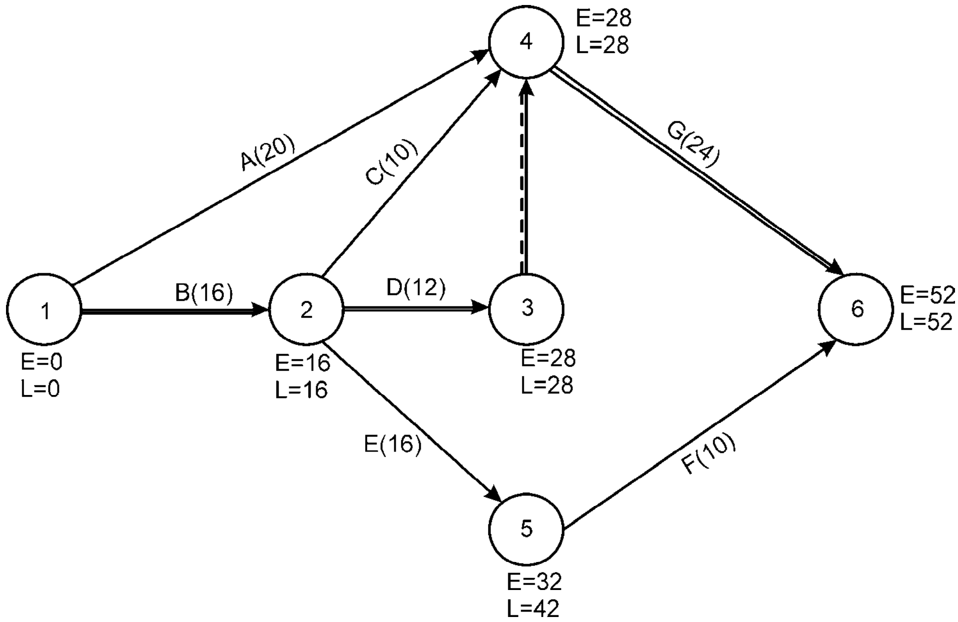


Figure 23

Path 1-2-3-4-6 is critical path.

Project duration is 52 weeks.

Total cost of the project = Total Direct Cost + Indirect Cost

$$= (\underbrace{40 + 30 + 16 + 22 + 18 + 10 + 6}_{\text{In thousands}}) + \underbrace{800}_{\text{Indirect cost per day}} \times \underbrace{7}_{\text{No. of days in a week}} \times \underbrace{52}_{\text{Project duration in weeks}}$$

= 1,42,000 + 2,91,200

= ₹ 4,33,200

Indirect cost is ₹ 800 per day or ₹ 5,600 per week.

We have to Crash the project step by step.

Activity	Normal Duration (Weeks)	Crash duration (Weeks)	Normal cost (₹'000)	Crash Cost (₹'000)	Cost Slope (₹'000)
A (1-4)	20	14	40	60	$\frac{60-40}{20-14} = 3.33$
B (1-2)	16	12	30	40	$\frac{40-30}{16-12} = 2.5$
C (2-4)	10	8	16	28	$\frac{28-16}{10-8} = 6$
D (2-3)	12	8	22	30	$\frac{30-22}{12-8} = 2$
E (2-5)	16	10	18	30	$\frac{30-18}{16-10} = 2$
F (5-6)	10	8	10	16	$\frac{16-10}{10-8} = 3$
G (4-6)	24	16	6	8	$\frac{8-6}{24-16} = 0.25$

The paths of the network are given below:

1-2-3-4-6 = 52 weeks

1-2-4-6 = 50 weeks

1-4-6 = 44 weeks

1-2-5-6 = 42 weeks

The critical activity G with cost slope of ₹ 250 (0.25 × 1,000) per week is least expensive and can be crashed by 8 weeks i.e., 24 weeks – 16 weeks.

With crashing of activity G, the new cost involved is

= Total Direct Cost + Increased Cost due to Crashing of G + Indirect cost 44 weeks.

= ₹ 1,42,000 + ₹ (250 × 8) + ₹ (800 × 7 × 44)

= ₹ 3,90,400

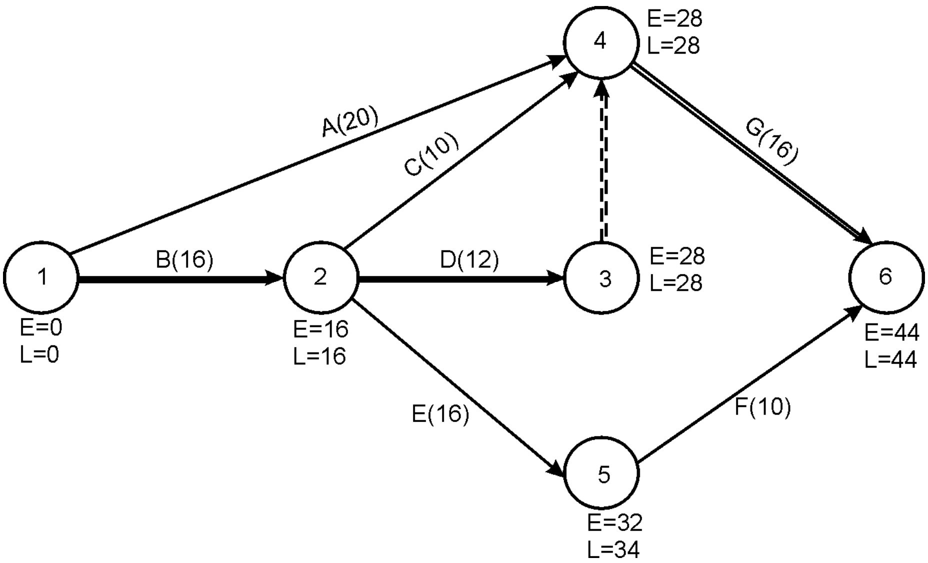


Figure 24

Various paths of the above network are given below:

1-2-3-4-6 = 44 weeks.

1-2-4-6 = 42 weeks

1-4-6 = 36 weeks

1-2-5-6 = 42 weeks

1-2-3-4-6 is still critical path

	B	D	Dummy	G
Activity	1-2	2-3	3-4	4-6
Slope	2.5	2	–	0.25
Can be Crashed by Max.	4 weeks (16-12)	4 weeks (12-8)	–	0 weeks (24-16-8*)

* Already crashed in first step

Since the least expensive activity on the critical path is D.

It can be crash by two week.

With crashing of activity D by two week, the new cost involved is = ₹ 1,42,000 + ₹ 2,000 + ₹ (2,000 × 2) + ₹ (5,600 × 42)

= ₹ 1,42,000 + 2,000 + ₹ (2,000 × 2) + ₹ (5,600 × 42)

= ₹ 3,83,200

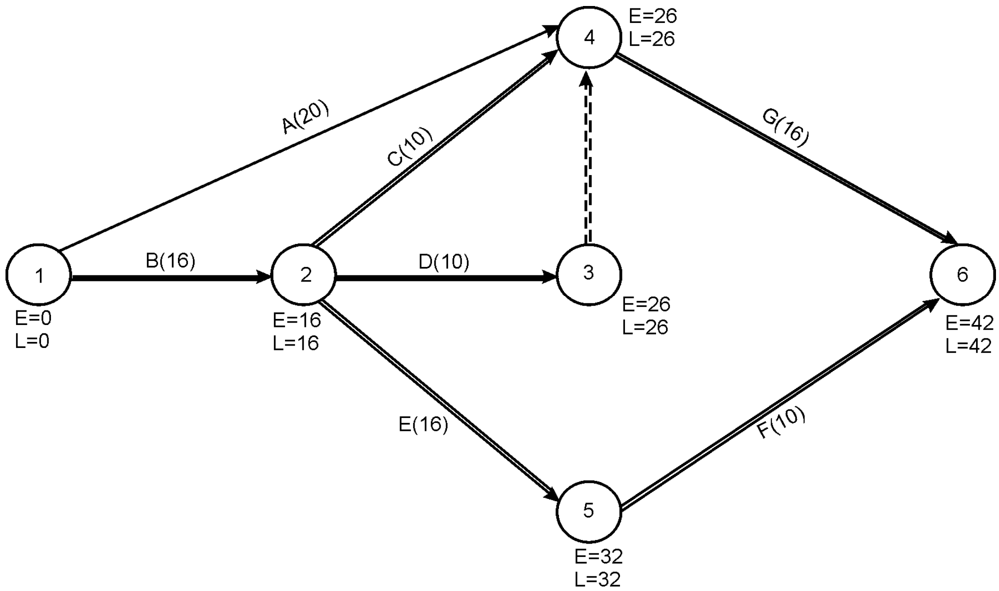


Figure 25

Various paths of the above network are given below :

1-2-3-4-6 = 42 weeks

1-2-4-6 = 42 weeks

1-4-6 = 36 weeks

1-2-5-6 = 42 weeks

Since activity B is common in all three critical paths it can be crashed to the maximum possible of 4 weeks i.e., 16 weeks-12 weeks.

The new total cost involved with duration of 38 weeks is as follows :

= ₹ 1,42,000 + ₹ 2,000 + ₹ 4,000 + ₹ (4 × 2,500) + ₹ (5,600 × 38)

= ₹ 3,70,800

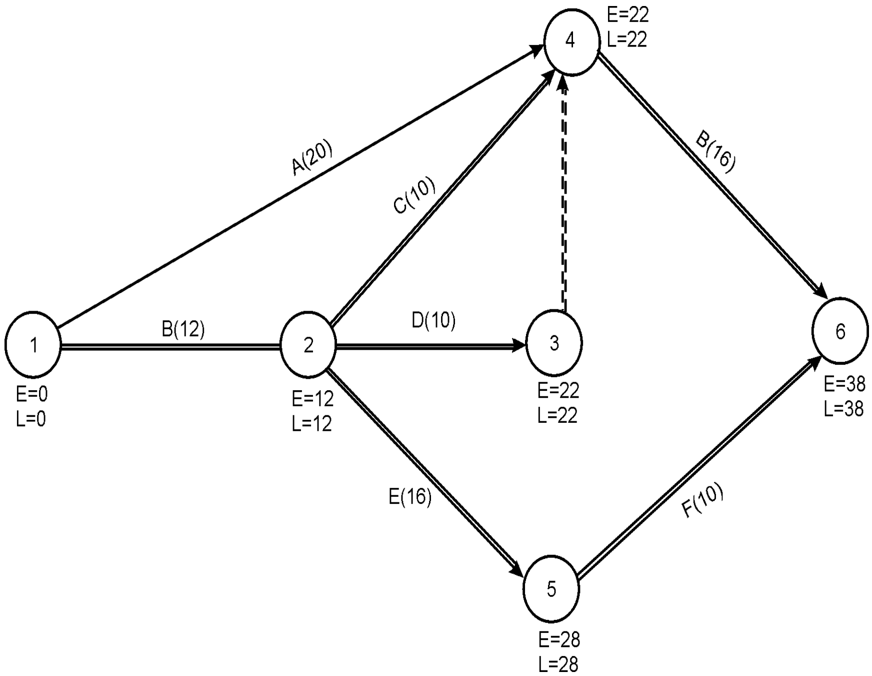


Figure 26

Various path of the above network are given below.

1-2-3-4-6	= 38 weeks
1-2-4-6	= 38 weeks
1-4-6	= 36 weeks
1-2-5-6	= 38 weeks

<u>1 – 2 – 3 – 4 – 6</u>

↓ ↓ ↓ ↘

B D Dummy G

B, G, cannot be further crashed.

However D can be crash by 2 weeks.

<u>1 – 2 – 4 – 6</u>

↓ ↓ ↓

B C G

B, G, cannot be further crashed.

However C can be crash by 2 weeks.

1 – 4 – 6 —

↓ ↓

A G

1 – 2 – 5 – 6 B cannot be further crashed.

↓ ↓ ↓ E can be crashed by. 2 weeks due to lowest cost slope in

B E F comparison with F.

Total New Cost.

= ₹ 1,42,000 + ₹ 2,000 + ₹ 4,000 + ₹ 10,000 + ₹ (2,000 × 2 + 6,000 × 2 + 2,000 × 2) + (₹ 5,600 × 36)

= ₹ 3,79,600

No benefit in crashing the project to 36 weeks as cost is increasing by 8,800/-.

Crashing Schedule of Project

Project duration weeks	Crashing activity and weeks	Direct Cost		Indirect Cost	Total (₹)
		Normal	Crashing		
52	—	1,42,000	—	2,91,200	4,33,200
44	G by 8 weeks	1,42,000	2,000	2,46,400	3,90,400
42	D by 2 weeks	1,42,000	2,000 + 4,000	2,35,200	3,83,200
38	B by 4 weeks	1,42,000	6,000 + 10,000	2,12,800	3,70,800
36	C by 2 weeks D by 2 weeks E by 2 weeks	1,42,000	16,000 + 4,000 12,000 + 4,000	2,01,600	3,79,600

Minimum Cost of the project is ₹ 3,70,800 at 38 weeks.

14.7 A Few Comments on Assumptions of PERT & CPM

1. Beta distribution may not always be applicable.

2. The formulae for the expected duration and S.D. are simplifications. Maccrinnon and Ryavec reached the conclusion that in certain cases the errors, because of these assumptions, may even be to the tune of 33%.

3. The errors owing to the aforesaid simplification and assumption may be compounded or may cancel each other to an extent.

4. In computing the S.D. of the critical path independence of activities is implied. Limitations of resources may invalidate the independence which exists by the very definition of an activity.

5. It may not always be possible to sort out completely identifiable activities and to state where they begin and where they end.

6. In projects fraught with in certainty it is natural that there exist alternatives with differing outcomes. For example, if a particular hardness is not obtained in a metal, an alloy might have to be used that is more expensive and also inferior on certain technical considerations. There have been theoretical developments in this regard, and it may be worthwhile to incorporate the concept of decision tree analysis depending upon the situation.

7. Time estimates have an element of subjectiveness and, to that extent, the techniques could be weak. The contractors react to this weakness shrewdly whilst bidding. If there are cost plus contracts they would deliberately “under estimate” the time for chances of being awarded with the contract. Incentive type contracts might lead to an opposite bias.

8. Cost-time tradeoffs, for deriving the cost curve slopes, to be discussed soon, are subjective again and call for a great deal of expertise of the technology as well as genuine effort to estimate. Often the engineers tend to be lax here; occasionally with the honest deliberation event, the guesses may be wide off the mark.

14.8 Distinction Between PERT and CPM

The PERT and CPM models are similar in terms of their basic structure, rationale and mode of analysis. However, there are certain distinctions between PERT and CPM networks which are enumerated below:

1. CPM is activity oriented i.e. CPM network is built on the basis of activities. Also results of various calculations are considered in terms of activities of the project. On the other hand, PERT is event oriented.

2. CPM is a deterministic model i.e. It does not take into account the uncertainties involved in the estimation of time for execution of a job or an activity. It completely ignores the probabilistic element of the problem. PERT, however, is a probabilistic model. It uses three estimates of the activity time; optimistic, pessimistic and most likely; with a view to take into account time uncertainty. Thus, the expected duration of each activity is probabilistic and expected duration indicates that there is fifty per cent probability of getting the job done within that time.

3. CPM places dual emphasis on time and cost and evaluates the trade-off between project cost and project time. By deploying, additional resources, it allows the critical

path project manager to manipulate project duration within certain limits so that project duration can be shortened at an optimal cost. On the other hand, PERT is primarily concerned with time. It helps the manager to schedule and coordinate various activities so that the project can be completed on scheduled time.

4. CPM is commonly used for those projects which are repetitive in nature and where one has prior experience of handling similar projects. PERT is generally used for those projects where time required to complete various activities are not known as priori. Thus, PERT is widely used for planning and scheduling research and development projects.